IDENTIFYING FACTORS THAT INFLUENCE SPORT GAMES IN MALAYSIA AND OLYMPIC HOSTS COUNTRIES

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Motivation

During 2020 Olympics Tokyo, Japan had shown the world that they are a strong competitor by getting ranked 3rd

This is the first time they ever reach the top three spots.

It piqued my interest in the hosting effect of Olympics.

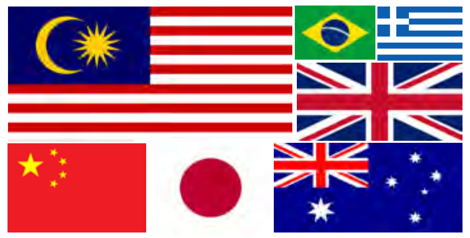
Research Question

1) What effect does the host country have in the medals won at Olympics?

2) Is the performance of countries in Olympic games affected by economic factors of the country?

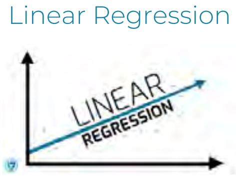
3) If Malaysia become a host, will it affect Malaysia's total medal won in Olympics?

Scope of Studies

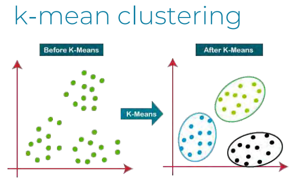


Method

Linear Regression



k-mean clustering



Materials

Olympics results are obtained through Wikipedia
Countries GDP are obtained through The World Bank

Literature Review

Host country have advantage when it comes to field familiarity (Simon Shibli & Jerry Bingham, 2008). Country with high GDP can affect the country's performance in Olympics(Andrew B. Bernard & Meghan R. Busse, 2004)

Results and Discussions

Hosting Effect

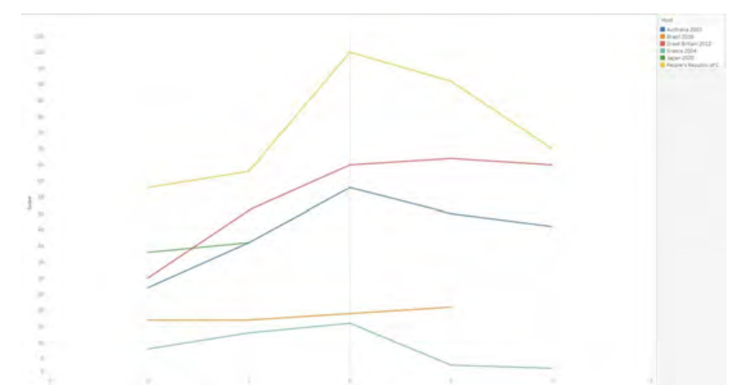


Figure 1 – Score of countries that have hosted Olympics

Year 0 = Year Hosted
Year 1 = Next Games
Year -1 = Previous Games

Gold = 3 points
Silver = 2 points
Bronze = 1 points

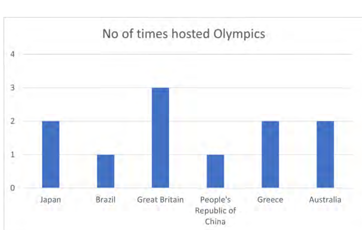


Figure 2

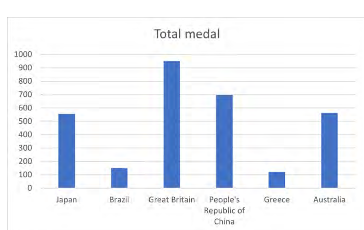


Figure 3

Greece hosted 2 times, but we exclude 1 because the one of it is the first ever Olympics. Assume Greece hosted once. Figure 2 reflect figure 3 except for China.

Economic Effect

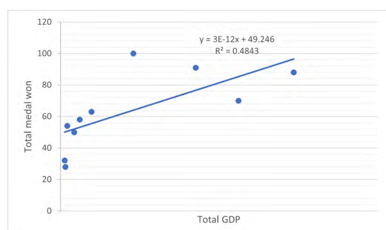


Figure 4 (China)
R = 0.6959

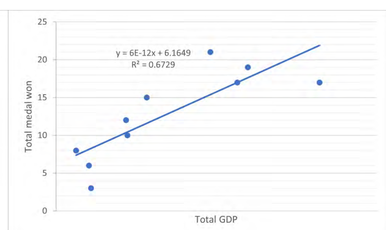


Figure 5 (Brazil)
R = 0.8203

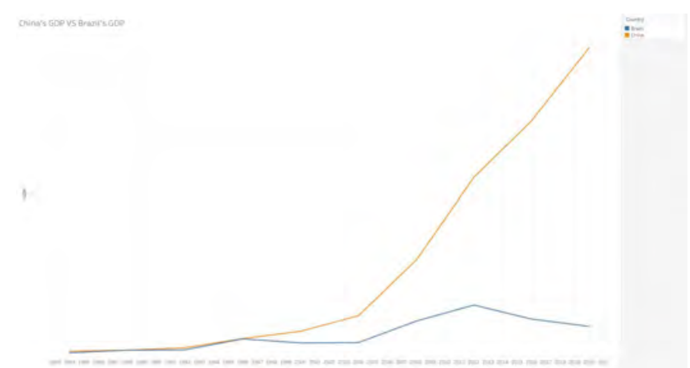
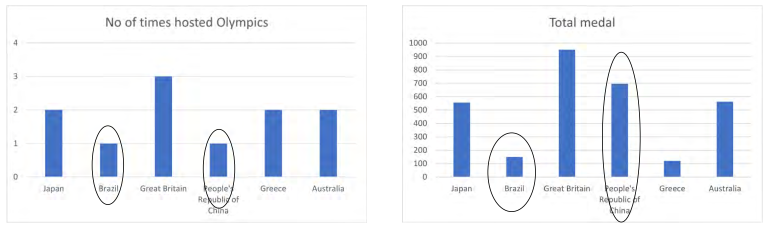


Figure 6 – China's GDP VS Brazil's GDP



China high medal count due to its GDP.

Application on Malaysia

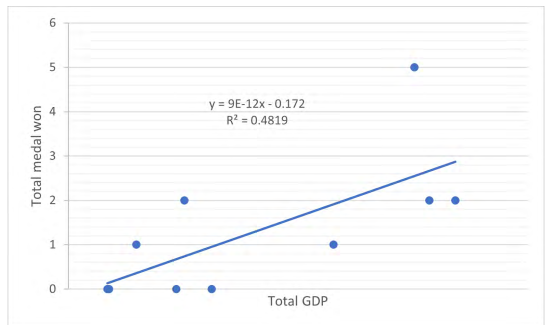
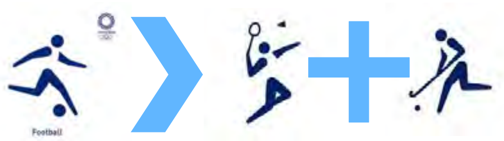


Figure 7 (Malaysia)
R = 0.6942

Malaysia could increase medal count if GDP increase



However, Malaysia prefer to invest money into football instead of other sports

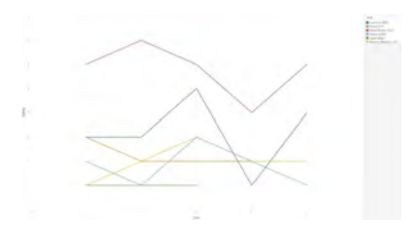
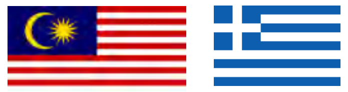


Figure 8 – Score of countries that have hosted Olympics (Sailing)

Sailing is one of the geography advantage that host can get, Including Malaysia.

Moral support from local supporter is also host advantage. Players will be more confident.



Malaysia is in the same group as Greece by k-mean

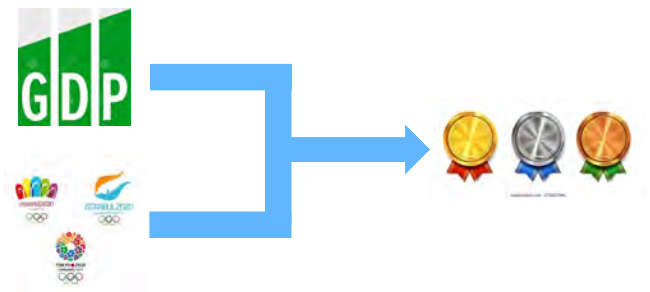
Speculation : Value by value : + 3 medals
Percentage : negligible (20% increase)

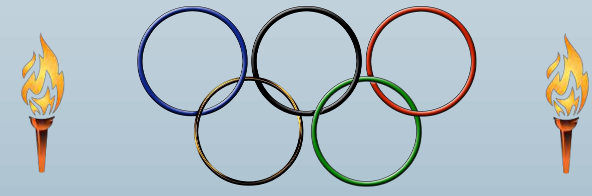
Conclusion

Hosting Olympics does make its host better at winning more medals.

Higher GDP make the country better at Olympics.

Malaysia could win more medals at Olympics by improving their economy and also become a host of Olympic Games.





Recommendation

Malaysia should focus on stabilizing their economic due to Covid-19 pandemic.

Invest more money into sports that does well internationally like badminton and field hockey.

References

Shibli, S., & Bingham, J. (2008). A forecast of the performance of China in the Beijing Olympic Games 2008 and the underlying performance management issues. Managing Leisure, 13(3–4), 272–292. https://doi.org/10.1080/13606710802200977

Bernard, A. B., & Busse, M. R. (2004). Who Wins the Olympic Games: Economic Resources and Medal Totals. Review of Economics and Statistics, 86(1), 413–417. https://doi.org/10.1162/003465304774201824

Gautam, A. (2021, December 15). Visual Analysis of Olympics Data - Towards Data Science. https://towardsdatascience.com/visual-analysis-of-olympics-data-16273f7c6cf2

LAGRANGE MECHANICS

another useful approach in finding an equation of motion

WHAT IS LAGRANGIAN

Lagrangian, L is defines as the different between the kinetic energy, T and potential energy, V

$$L = T - V$$

The equation of motion can be determined by using the Euler-Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

In calculating the Lagrangian, the path that will be taken by the system is actually the path that minimized the action.
The action is defined as the intergral of the Lagrangian over time

$$S \equiv \int_{t_1}^{t_2} L(x, \dot{x}, t)\, dt$$

NEWTONIAN

In classical mechanics, we can find the equation of motion of a system by using the Newton Second Law of Motion

F=ma

That is, an object acceleration is determined by two variables that is the net force action on the object and the mass of the object.

PRINCIPLE OF LEAST ACTION

In a physical system, the paths followed by different part of a system are those that minimized the action. It is said to be stationary path or stationary functions of a functions.

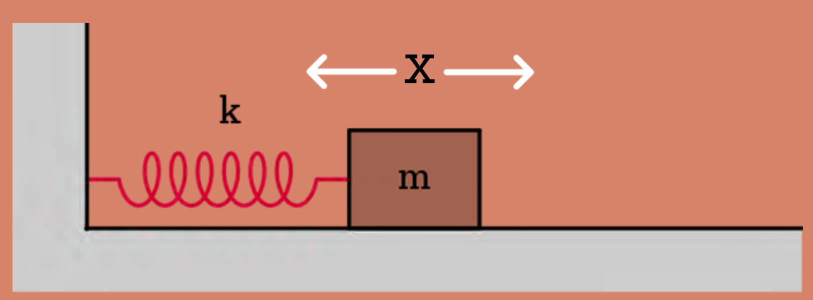
It is a variational principle to the action of a mechanical system, then the solution for the mechanical system will follow the path of least action. This will yields to the equation fo motion of the system

This means the minimum of the action (integral of the Lagrangian) will give rise to a set of equation of motions for the system and this will give us the Euler-Lagrange Equation. This can be derive by using the Calculus of Variations to find a stationary functions of a functions

LAGRANGIAN VS NEWTONIAN

- The lagrangian methods actually gives us the equation of motion without considering forces at all. We only need to consider energy.
- Besides, we don't need to involve vector in finding an equation of motions since energy is a scalar.
- •It will become easier to use the Euler-Lagrange equation because they are very good at dealing with multiple coordinates and they actually gave us the equation of motion for each coordinate.

EXAMPLE



The Lagrange is given by,

$$L = T - V$$

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

The Euler-Lagrange equation is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}.$$

We have,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}(m\dot{x}) = m\ddot{x}$$

and

$$\frac{\partial L}{\partial x} = -kx.$$

Hence, we have

$$m\ddot{x} = -kx$$

Essentially, the net force of our system

$$F = ma$$

is equal to the force exerted by the spring that is

$$F = ma = m\ddot{x} = -kx$$

Hence, the equation of motion of the system is

$$\ddot{x} = -\frac{k}{m}x$$

where $\ddot{x}$ is an acceleration.

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Title:A Study of Additive Maps on Real Invertible Matrices

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Chapter 1

Introduction

This mathematical science project is devoted the study of additive maps on real invertible matrices.

Chapter 2

Notation and Examples

Example 2.0.1 *Let $n \geqslant 3$ be an integer and let $\psi_1 : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be the map defined by*

$$\psi_1(A) = \det(A) \quad \text{for all } A \in M_n(\mathbb{R}),$$

where $\det(A)$ denotes the determinant of the matrix A. We first note that $\det(A) = 0$ for all rank one matrices. Let $A, B \in M_n(\mathbb{R})$ be of rank one matrices. Note that $\operatorname{rank}(A+B) \leqslant \operatorname{rank}(A)+\operatorname{rank}(B) = 2$. Then $\psi_1(A) = 0$, $\operatorname{rank}(B) = 0$ and $\psi_1(A+B) = 0$. So

$$\psi_1(A+B) = \psi_1(A) + \psi_1(B)$$

for all rank one matrices $A, B \in M_n(\mathbb{R})$.

However, ψ_1 is not an additive map. To see this, let $X_0 = E_{11}$ and $Y_0 = I_n - E_{11}$, where I_n is the identity matrix. Then $\psi_1(X_0+Y_0) = \det(X_0+Y_0) = \det(I_n) = 1$, but $\psi_1(X_0)+\psi_1(Y_0) = 0$ because $\psi_1(X_0) = \psi_1(Y_0) = 0$. Hence $\psi_1(X_0+Y_0) \neq \psi_1(X_0)+\psi_1(Y_0)$, and hence ψ_1 is not an additive map.

Example 2.0.2 *Let $n \geqslant 4$ be an integer and let $\psi_2 : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be the map defined by*

$$\psi_2(A) = \operatorname{adj}(A) \quad \text{for all } A \in M_n(\mathbb{R}).$$

Here, $\operatorname{adj}(A)$ denotes the adjoint of the matrix A, see [1, Section 2.3].

We first show that $\psi_2(A+B) = \psi_2(A)+\psi_2(B)$ for all rank one matrices $A, B \in M_n(\mathbb{R})$. Let $A, B \in M_n(\mathbb{R})$ be of rank one. Then $\operatorname{rank}(A+B) \leqslant \operatorname{rank}(A)+\operatorname{rank}(B) = 2$. Since $n \geqslant 4$, we see that $\operatorname{adj}(A) = 0$, $\operatorname{adj}(B) = 0$ and $\operatorname{adj}(A+B) = 0$. Consequently, we obtain

$$\psi_2(A+B) = 0 = \psi_2(A) + \psi_2(B).$$

Notice that ψ_2 is not an additive on $M_n(\mathbb{R})$. let $A_0 = E_{11} + E_{22}$ and $B_0 = I_n - A_0$. Then $\operatorname{rank}(A_0) = 2 \leqslant n-2$ and $\operatorname{rank}(B_0) = n-2$. So $\psi_2(A_0) = 0 = \psi_2(B_0)$. However, $\psi_2(A_0+B_0) = \psi_2(I_n) = \operatorname{adj}(I_n) = I_n$. Hence $\psi_2(A_0+B_0) \neq \psi_2(A_0) + \psi_2(B_0)$ and ψ_2 is not an additive map.

Example 2.0.3 *Let $n \geqslant 3$ be an integer and let $\psi_3 : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be the map defined by*

$$\psi_3(A) = \begin{cases} A & \text{if } A \text{ is singular,} \\ 0 & \text{if } A \text{ is invertible.} \end{cases}$$

Let $A, B \in M_n(\mathbb{R})$ be rank one matrices. Then A, B and $A+B$ are singular matrices, and so $\psi_3(A+B) = 0 = \psi_3(A)+\psi_3(B)$. Consequently,

$$\psi_3(A+B) = \psi_3(A) + \psi_3(B)$$

for all rank one matrices $A, B \in M_n(\mathbb{R})$. We now claim that ψ_3 is not an additive map. Let $H_0 = E_{11}$ and $K_0 = E_{22} + \cdots + E_{nn}$. Then H_0 and K_0 are singular and $H_0 + K_0 = I_n$. Hence $\psi_3(H_0+K_0) = \psi_3(I_n) = I_n \neq 0 = \psi_3(H_0) + \psi_3(K_0)$.

Examples 2.0.1, 2.0.2 and 2.0.3 give some examples of non-additive maps $\psi_i : M_n(\mathbb{R}) \to M_n(\mathbb{R})$, $i = 1, 2, 3$, satisfying

$$\psi_i(A+B) = \psi_i(A) + \psi_i(B)$$

for all rank one matrices $A, B \in M_n(\mathbb{R})$. This shows the indispensability of invertible condition in our study.

Chapter 3

Preliminary Results

We first establish some important results for the study of additive maps on invertible matrices.

Lemma 3.0.1 **(Rank Factorization)** *Let $n \geqslant 2$ and $1 \leqslant r \leqslant n$ be integers. Then $A \in M_n(\mathbb{R})$ is of rank r if and only if there exist invertible matrices $P, Q \in M_n(\mathbb{R})$ such that*

$$A = P \begin{pmatrix} I_r & 0 \\ 0 & 0_{n-r} \end{pmatrix} Q.$$

Proof. The sufficiency is clear. Consider the necessity. Let $A = (a_{ij}) \in M_n(\mathbb{R})$. Since A is nonzero, there exist $1 \leqslant i_0 \leqslant m$ and $1 \leqslant j_0 \leqslant n$ such that $a_{i_0j_0} \neq 0$. Consider

$$A \xrightarrow[C_1 \leftrightarrow C_{j_0}]{R_1 \leftrightarrow R_{i_0}} \begin{pmatrix} a_{i_0j_0} & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{pmatrix} \xrightarrow{R_1 \to a_{i_0j_0}^{-1} R_1} \begin{pmatrix} 1 & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{pmatrix} = A_1.$$

Let $A_1 = (d_{ij}) \in M_n(\mathbb{R})$. We next consider

$$A_1 \xrightarrow[j=2,\ldots,n]{C_j \to C_j - d_{1j}C_1} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ * & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & * \end{pmatrix} \xrightarrow[i=2,\ldots,n]{R_i \to R_i - d_{i1}R_1} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & A_2 & \\ 0 & & & \end{pmatrix},$$

where $A_2 \in M_{n-1}(\mathbb{R})$. Then $A \sim I_1 \oplus A_2$. If $\operatorname{rank} A = 1$, then $A_2 = 0$ and $A \sim E_{11}$, that is, there exist invertible matrices $P_1 \in M_n(\mathbb{R})$ and $Q_1 \in M_n(\mathbb{R})$ such that $A = P_1E_{11}Q_1$. We are done.

If $\operatorname{rank} A = r > 1$, then $A_2 \neq 0$ and we continue this process on A_2 and obtain

$$I_1 \oplus A_2 \sim I_1 \oplus \begin{pmatrix} I_{r-1} & 0 \\ 0 & 0 \end{pmatrix},$$

and so

$$A \sim \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}.$$

Thus there are invertible $P, Q \in M_n(\mathbb{R})$ such that $A = P(I_r \oplus 0)Q$. □

Lemma 3.0.2 *Let $\psi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a map satisfying $\psi(A+B) = \psi(A)+\psi(B)$ for all invertible $A, B \in M_n(\mathbb{R})$. Then $\psi(0) = 0$ and $\psi(-A) = -\psi(A)$ for all invertible $A \in M_n(\mathbb{R})$.*

Proof. We first claim that $\psi(-A) = -\psi(A)$ for every invertible matrix $A \in M_n(\mathbb{R})$. Let $A \in M_n(\mathbb{R})$ be invertible. Note that

$$\begin{aligned} \psi(A) &= \psi(2A - A) \\ &= \psi(2A + (-A)) \\ &= \psi(2A) + \psi(-A) \\ &= (\psi(A + A)) + \psi(-A) \\ &= 2\psi(A) + \psi(-A). \end{aligned}$$

Then $\psi(_A) = -\psi(A)$ for all invertible matrices $A \in M_n(\mathbb{R})$. Let $A \in M_n(\mathbb{R})$. We see that

$$\psi(0) = \psi(A - A) = \psi(A + (-A)) = \psi(A) + \psi(-A) = \psi(A) - \psi(A) = 0.$$

This completes our proof. □

Lemma 3.0.3 *Let $\psi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a map satisfying $\psi(A+B) = \psi(A)+\psi(B)$ for all invertible $A, B \in M_n(\mathbb{R})$. Then the following hold.*

(a) *$\psi(A - B) = \psi(A) - \psi(B)$ for all invertible $A, B \in M_n(\mathbb{R})$.*

(b) *Let $A, B \in M_n(\mathbb{R})$ be such that A and $A + B$ invertible. Then $\psi(A + B) = \psi(A) + \psi(B)$.*

Proof. (a). Let $A, B \in M_n(\mathbb{R})$ be invertible. Then $\psi(A - B) = \psi(A + (-B)) = \psi(A) + \psi(-B) = \psi(A) - \psi(B)$ by Lemma 3.0.3.

(b) Note that $\psi(A+B) - \psi(A) = \psi((A+B) - A) = \psi(B)$ by (a). Hence $\psi(A + B) = \psi(A) + \psi(B)$. □

We now prove a technical key result.

Lemma 3.0.4 *Let $1 \leqslant k \leqslant n$ and let $A, B \in M_n(\mathbb{R})$ be matrices such that $\operatorname{rank}(A) = k$ and $\operatorname{rank}(B) = 1$. Then there exist an invertible matrices $P, Q \in M_n(\mathbb{R})$ such that $PAQ = I_k \oplus 0_{n-k}$ and PBQ is one of the following forms:*

$$aE_{11}, \quad aE_{12}, \quad E_{1,k+1}, \quad E_{k+1,1}, \quad E_{k+1,k+1}$$

for some nonzero $a \in \mathbb{R}$.

Lemma 3.0.5 *Let $\psi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a map satisfying $\psi(A+B) = \psi(A)+\psi(B)$ for all invertible $A, B \in M_n(\mathbb{R})$. Then*

$$\psi(X + Y) = \psi(X) + \psi(Y)$$

for every $X \in M_n(\mathbb{R})$ and rank one $Y \in M_n(\mathbb{R})$.

Proof. Let $X, Y \in M_n(\mathbb{R})$ be matrices such that $\operatorname{rank}(Y) = 1$. If $X = 0$, then $\psi(X) = 0$ by Lemma 3.0.2, and $\psi(X + Y) = \psi(Y) = \psi(X) + \psi(Y)$. Consider now $X \neq 0$. Let $\operatorname{rank}(X) = k$. By Lemma 3.0.4, there exist invertible matrices $P, Q \in M_n(\mathbb{R})$ such that $PXQ = I_k \oplus 0_{n-k}$ and PYQ is one of the following forms:

$$a_0E_{11}, \quad a_0E_{12}, \quad E_{1,k+1}, \quad E_{k+1,1}, \quad E_{k+1,k+1}$$

for some nonzero scalar $a_0 \in \mathbb{R}$. Let $b \in \mathbb{R}$ be a nonzero scalar such that $b \notin \{-1, 1, a_0\}$. We set

$$Z = P^{-1}(bI_n)Q^{-1}.$$

Then $X + Z = P^{-1}((1 + b)I_k \oplus I_{n-k})Q^{-1}$ and $Y - Z$ is one of the following forms:

$$P^{-1}(a_0E_{11} - bI_n)Q^{-1}, \quad P^{-1}(a_0E_{12} - bI_n)Q^{-1}, \quad P^{-1}(E_{1,k+1} - bI_n)Q^{-1},$$

$$P^{-1}(E_{k+1,1} - bI_n)Q^{-1}, \quad P^{-1}(E_{k+1,k+1} - bI_n)Q^{-1}.$$

Then $Z, -Z, X + Z$ and $Y - Z$ are invertible. By the hypothesis, we have

$$\psi(X+Y) = \psi(X+(Z-Z)+Y) = \psi((X+Z)+(Y-Z)) = \psi(X+Z)+\psi(Y-Z).$$

Moreover, since $X + Z$ and $Y - Z$ are invertible, it follows from Lemma 3.0.3(a) that

$$\psi(X) = \psi((X + Z) - Z) = \psi(X + Z) - \psi(Z).$$

Hence $\psi(X+Z) = \psi(X)+\psi(Z)$. Likewise, since Z and $Y-Z$ are invertible, it follows from Lemma 3.0.3(b) that $\psi(-Z + Y) = \psi(-Z) + \psi(Y)$. Hence $\psi(Y - Z) = \psi(Y) - \psi(Z)$ by Lemma 3.0.2. Consequently,

$$\begin{aligned} \psi(X + Y) &= \psi(X + Z) + \psi(Y - Z) \\ &= (\psi(X) + \psi(Z)) + (\psi(Y) - \psi(Z)) \\ &= \psi(X) + \psi(Y). \end{aligned}$$

The proof is completed. □

Chapter 4

Main Results

We are now ready to prove the main result of this project.

Theorem 4.0.1 *Let $n \geqslant 2$ be an integer and let $\psi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a map. Then $\psi(A + B) = \psi(A) + \psi(B)$ for all invertible $A, B \in M_n(\mathbb{R})$ if and only if ψ is an additive map*

Proof. The sufficiency is clear. For the necessity, we let $A, B \in M_n(\mathbb{R})$ be matrices. Note that if $B = 0$, then $\psi(B) = 0$ by Lemma 3.0.2, and

$$\psi(A + B) = \psi(A) = \psi(A) + \psi(B).$$

The result holds. Consider $B \neq 0$. Let $\operatorname{rank}(B) = r$. By Lemma 3.0.1, there exist invertible matrices $P, Q \in M_n(\mathbb{R})$ such that

$$B = P(I_r \oplus 0_{n-r})Q = P(E_{11} + \cdots + E_{rr})Q = B_1 + \cdots + B_r$$

where $B_i = PE_{ii}Q$ and $\operatorname{rank}(B_i) = 1$ for $i = 1, \ldots, r$. Then

$$\begin{aligned} \psi(A + B) &= \psi(A + (B_1 + \cdots + B_r)) \\ &= \psi((A + B_1 + \cdots + B_{r-1}) + B_r) \\ &= \psi(A + B_1 + \cdots + B_{r-1}) + \psi(B_r) \end{aligned}$$

by Lemma 3.0.5. Continuing in this fashion, we obtain

$$\psi(A + B_1 + \cdots + B_{r-1}) = \psi(A) + \psi(B_1) + \cdots + \psi(B_{r-1}),$$

and so $\psi(A + B) = \psi(A) + \psi(B_1) + \cdots + \psi(B_r)$. Again, by Lemma 3.0.5, we see that

$$\begin{aligned} \psi(B_1) + \cdots + \psi(B_r) &= (\psi(B_1) + \psi(B_2)) + \psi(B_3) \cdots + \psi(B_r) \\ &= \psi(B_1 + B_2) + \psi(B_3) + \cdots + \psi(B_r) \\ &= (\psi(B_1 + B_2) + \psi(B_3)) + \psi(B_4) + \cdots + \psi(B_r) \\ &= \psi(B_1 + B_2 + B_3) + \psi(B_4) + \cdots + \psi(B_r) \end{aligned}$$

Continuing in this fashion, we obtain $\psi(B_1)+\cdots+\psi(B_r) = \psi(B_1+\cdots+B_r) = \psi(B)$. Consequently,

$$\psi(A+B) = \psi(A)+\psi(B_1)+\cdots+\psi(B_r) = \psi(A)+\psi(B_1+\cdots+B_r) = \psi(A)+\psi(B).$$

Hence ψ is an additive map on $M_n(\mathbb{R})$. The proof is completed. □

We end the project with an application of Theorem 4.0.1. It starts with a well known structural result of commuting additive maps in the study of functinal identity by Bresar in [2, Theorem 3.2].

Lemma 4.0.2 *Let $n \geqslant 2$ be an integer and let $\psi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be an additive map. Then $\psi(A)A = A\psi(A)$ for all $A \in M_n(\mathbb{R})$ if and only if there exit $\lambda \in \mathbb{R}$ and an additive map $\mu : M_n(\mathbb{R}) \to \mathbb{R}$ such that*

$$\psi(A) = \lambda A + \mu(A)I_n$$

for all $A \in M_n(\mathbb{R})$.

We are now ready to give an application of Theorem 4.0.1 to the study of functional identity on real matrices.

Theorem 4.0.3 *Let $n \geqslant 2$ be an integer and let $\psi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ be a map. Then*

$$\psi(A + B)(A + B) = (A + B)(\psi(A) + \psi(B))$$

for all $A, B \in M_n(\mathbb{R})$ if and only if there exit $\lambda \in \mathbb{R}$ and an additive map $\mu : M_n(\mathbb{R}) \to \mathbb{R}$ such that

$$\psi(A) = \lambda A + \mu(A)I_n$$

for all $A \in M_n(\mathbb{R})$.

Proof. Consider the sufficiency. Note that

$$\begin{aligned} \psi(A + B)(A + B) &= (\lambda(A + B) + \mu(A + B)I_n)(A + B) \\ &= \lambda A^2 + \lambda B^2 + \lambda AB + \lambda BA + \mu(A)A \\ &\quad + \mu(B)B + \mu(A)B + \mu(B)A \\ &= A\lambda A + A\mu(A) + A\lambda B + A\mu(B) \\ &\quad + B\lambda A + B\mu(A) + B\lambda B + B\mu(B) \\ &= (A + B)(\psi(A) + \psi(B)) \end{aligned}$$

for all $A, B \in M_n(\mathbb{R})$.

For the necessity, we first show that $\psi(0) = 0$. Note that

$$\psi(I_n) = \psi(I_n + 0)(I_n + 0) = (I_n + 0)(\psi(I_n) + \psi(0)) = \psi(I_n) + \psi(0).$$

Then $\psi(0) = 0$. It follows that

$$\psi(A)A = \psi(A + 0)(A + 0) = (A + 0)(\psi(A) + \psi(0)) = A\psi(A)$$

for all $A \in M_n(\mathbb{R})$.

We show that $\psi(2A) = 2\psi(A)$ and $\psi(-A) = -\psi(A)$ for all invertible $A \in M_n(\mathbb{R})$. Let $A \in M_n(\mathbb{R})$ be invertible. Note that $2A\psi(2A) = \psi(2A)(2A) = \psi(A+A)(A+A) = (A+A)(\psi(A)+\psi(A)) = 2A(2\psi(A))$ yields $\psi(2A) = 2\psi(A)$ because $2A$ is invertible. Next, $A\psi(A) = \psi(A)A = \psi(-A+2A)(-A+2A) = (-A+2A)(\psi(-A)+\psi(2A)) = A(\psi(-A)+2\psi(A))$ leads to $\psi(-A) = -\psi(A)$. Finally we show that $\psi(A + B) = \psi(A) + \psi(B)$ for all invertible $A, B \in M_n(\mathbb{R})$. Let $A, B \in M_n(\mathbb{R})$ be invertible matrices. We see that

$$\begin{aligned} A\psi(A) &= \psi(A)A \\ &= \psi((A + B) + (-B))((A + B) + (-B)) \\ &= ((A + B) + (-B))(\psi(A + B) + \psi(-B)) \\ &= A(\psi(A + B) - \psi(B)). \end{aligned}$$

Then $\psi(A + B) = \psi(A) + \psi(B)$ for all invertible matrices $A, B \in M_n(\mathbb{R})$. Then ψ is an additive map by Theorem 4.0.1. Consequently, the result follows immediately from Lemma 4.0.2. □

Bibliography

[1]H.AntonC.Rorres,A.Kaul,Elementary Linear Algebra,12th edition,Wiley,USA,2019.

[2]M.Bresar. Centralizing mappings and derivations on prime rings.J.Algebra,156(1993),385-394.

[3]S.H.Friedberg,A.J. Insel,L.E.Spence Linear Algebra,4th edition,Prentice-Hall,Inc.,New Jersey,2003.

[4]G.Strang. Introduction to Linear Algebra,5th edition,Wellesley-Cambridge Press,USA2016.

[5]XXuH.Liu,Additive maps on rank-s matrices,Linear and Multilinear Algebra65(2017)806-812.

UNIVERSITI MALAYA

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SIN3015 Mathematical Science Project

DEVELOPING A PREDICTIVE MODEL FOR LAMB CARCASE C-SITE FAT DEPTH USING SUPPORT VECTOR MACHINE

Abstract *- This project was intended to obtain a good predictive model for the lamb carcase C-site fat depth using the Support Vector Machine(SVM) algorithm. We will then compare the result with the conventional multiple linear regression(MLR). 5-fold cross validation will be used to obtain the accuracy metrics. Python language and the scikit-learn library in conducting the analysis. At the end of this research, we found that the Linear Kernel in SVM fitted by the Principal Analysis Component (PCA) transformed data performed the best among all of the models. The SVM model also performs better than the MLR model.*

Objective *- This project aims to develop a prediction model for the lamb C-site fat depth using support vector machine. We also aim to compare the performance between Machine Learning(ML) algorithm and conventional statistical method.*

Introduction

Carcass refers to the body of a dead animal, especially a large one that is soon to be cut up as meat or eaten by wild animals. As the fat depth increase, the trimming cost increase causing the decrease in the economic value of the meat obtained. A group of researchers from the **Murdoch University** proposed a new technique in measuring the lamb C-site fat depth using microwave non-invasive technique. If we are able to develop an efficient predictive model to predict the fat depth on the livestock using microwave signal, we can optimize the economic value of the meat obtained.

Figure 1 : Measuring fat depth using Microwave Device

Unsupervised Learning

Principal Component Analysis (PCA)

- An efficient approach used in exploratory data analysis and predictive model development.
- Looking for new variables which are linear function that successively maximize variation and no autocorrelation with each other
- PCA allows us to reduce the number of features from 311 to 16.

K-mean Clustering

- A method of vector quantization that aim to partition n observations into k clusters
- Data points are gradually clustered based on similar characteristics by minimizing the sum of distances between the data points and the cluster centroid
- K-mean clustering allows us to reduce the number of features from 311 to 3 by using Elbow Method

Supervised Learning

Support Vector Machine (SVM)

- SVM analysis is a popular machine learning tool for classification and regression.
- It is a non-parametric algorithm as it is dependent on the kernel functions.
- Hyper-parameter tuning can help in optimizing the prediction.
- It is robust to outliers and having has excellent generalization capability with high prediction accuracy.

Multiple Linear Regression (MLR)

- MLR is a statistical method that is used to determine the relationship between variable.
- It helps us to determine how strong the relationship is between two or more independent variables and one dependent variable.
- 5 preliminary assumptions are associated with MLR which are the Linearity, Independence, Normality, Homoscedasticity and Multi-collinearity.

Support Vector Machine

Result on Different Kernel Used

From the result, we can see that the **SVM model with Linear Kernel fitted using PCA transformed data** was having the lowest RMSE and MAE and highest R-squared and adjusted R-squared among all of the models, and hence it performed the best among all of the models.

	Kernel	RMSE	MAE	R^2	Adj R^2
PCA	RBF	1.34	1.02	0.56	0.38
	Linear	1.12	0.87	0.62	0.56
	Poly	1.31	0.99	0.48	0.40
	Sigmoid	1.85	1.29	0.16	0.04

	Kernel	RMSE	MAE	R^2	Adj R^2
K-mean	RBF	1.25	0.96	0.52	0.51
	Linear	1.39	1.03	0.41	0.40
	Poly	1.21	0.94	0.55	0.54
	Sigmoid	1.74	1.39	-0.02	-0.04

Hyper-parameter Tuning

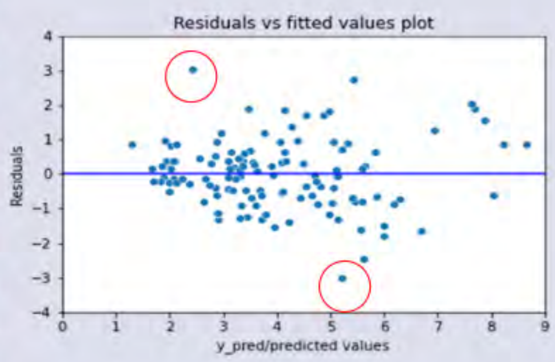
	C	RMSE	MAE	R^2	Adj R^2
PCA	1	1.12	0.87	0.62	0.56
	0.0899	1.11	0.86	0.62	0.57

As linear kernel is used, we will only consider the **cost parameter, C**. After tuning the value of C from the default value, 1 to 0.0899, we can observe a slightly improvement of the model performance with a lower RMSE and MAE and higher R-squared and adjusted R-squared.

Multiple Linear Regression

Assumptions Checking

The linearity, independence, normality and constant variance assumptions are fulfilled based on the plots. No multicollinearity issue is expected as PCA is used for dimensionality reduction. However there are **2 potential outliers** observed. We can further determine if the points are influential point by using the leverage and cook statistics and remove them.



Comparison of MLR and SVM

Method	RMSE	MAE	R^2	Adj R^2
SVM	1.11	0.86	0.62	0.57
MLR	1.14	0.89	0.60	0.54

§We can see that the Linear Kernel SVM model perform slightly better than the MLR model. §However, even though the result are similar, the result obtained from the MLR might not be accurate due to the existence of outliers. Further diagnostic on the outliers are needed.

Equation of the final model (SVM)

$$y = 3.9382 + (-0.5339)W_1 + (0.1891)W_2 + (0.6593)W_3 + (-0.1987)W_4 + (0.0315)W_5 + (0.2599)W_6 + 0.2775W_7 + (-0.3519)W_8 + (0.0555)W_9 + (0.4578)W_{10} + (-0.1221)W_{11} + (0.1042)W_{12} + (0.0416)W_{13} + (0.2611)W_{14} + (-0.2706)W_{15}$$

Conclusion *- We can develop an efficient predictive model for the lamb carcase C-site fat depth by using Support Vector Machine algorithm with linear kernel and PCA transformation. The ML algorithm seems to be more useful than the conventional statistical method in this case as the ML algorithm is easier in terms of application.*

Acknowledgement *- I would like to thank Dr. Elayaraja and Dr. Nur Anisah for their guidance and support throughout this project.*

UNIVERSITY OF MALAYA

SIN3015 Mathematical Science Project
On Squares in Finite Groups

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1) INTRODUCTION

- Let G be a finite group. The square of G is the set
$$G^2 = \{g \in G \mid \text{there exists } h \in G \text{ such that } g = h^2\}.$$
- We may also present it as
$$G^2 = \{g^2 \mid g \in G\}.$$
- The probability that a randomly chosen element in G has a square root is defined as
$$p(G) = \frac{|G^2|}{|G|}.$$
- It is always true that
$$\frac{1}{|G|} \leq p(G) \leq 1.$$

2) OBJECTIVES

- Investigate properties of the set of squares of G.
- Study the probability that a randomly chosen element in G has a square root.
- Survey some common patterns in that probability.

3) SOME PROPERTIES OF SQUARES OF FINITE GROUPS

- The **number of distinct squares** in an **odd ordered group** is the **same as the order of the group.**
- Any **homomorphic image of a square** is a **square.**
- The **conjugate of a square** is a **square.**
- Any **element of odd order** in a group **has a square root.**
- The **square of the group *G*** is a **subgroup of *G*** if *G* is an **abelian group**.

4) PROBABILITY THAT AN ELEMENT IN A GROUP HAS A SQUARE ROOT

Let *G* be a **finite group.** Then
- ***p(G) = 1/|G|*** if and only if *G* is an **elementary abelian *2*-group**;
- ***p(G) = 1*** if and only if **|*G*| is odd.**

Let *G* be a **finite abelian group.** Then
- ***p(G) = 1/(1+t(G))***, where *t(G)* is the number of involutions of *G;*
- ***p(G)* ≤ 1/2** if and only if **|*G*| is even.**

5) SOME FINDINGS

Let *n* be a positive integer. Then

(i) $p(C_{2n}) = \frac{1}{2}$;

(ii) $p(D_{2n}) = \frac{1}{4}$;

(iii) $p(Dic_{2n}) = \frac{1}{4}$;

(iv) $p(Q_{8n}) = \frac{1}{4}$.

G	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

6) REFERENCES

[1] Lucido, M. S. & Pournaki, M. R. (2005). Elements with square roots in finite groups. *Algebra Colloquium, 12*(4), 677-690.
[2] Jean B. Nganou. (2012). How rare are subgroups of index 2? *Mathematics Magazine, 85*(3), 215-220.
[3] *Small finite groups and cayley tables*, http://escarbille.free.fr/group.php, Last accessed: 14.11.2021.

A SURVEY ON LORENTZ TRANSFORMATION AND THE SPECIAL THEORY OF RELATIVITY

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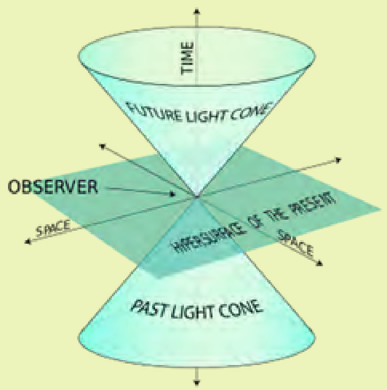
Galileo Galilei discovered that laws of mechanics are the same for all observers. So, given an observation by one observer, one could translate the observation to another observer by the Galilean transformation.

Mathematicians: Gauss, Riemann, Lorentz, Poincare, just to mention a few of them assume that the reality should not only have more dimensions, but also should be endowed with a curvature. The curvature should be positive, like of the sphere, or negative like of the hyperbolic space. The starting point of this understanding of this connection belongs to the physicist Albert Einstein, who invented theories of special and general relativity.

As the first step, our 3-d space and time were not anymore thought as two separated components, but they became a part of a bigger space. Namely, they formed the space-time, or a 4-d space which was furnished with special tool of measurement. The distance between two points in the space-time was no longer given by the Euclidean distance, or its metric, but it become a byproduct of the minkowski metric. The transformation between two observers was changed from Galilean transformation to the Lorentzian transformation to preserve the new type of metric in the universe.



Minkowski Spacetime

The Minkowski spacetime is a 4-dimensional real vector space $\mathcal{M}$ on which a non-degenerate, symmetric, bilinear form g of index 1 is defined. The points of $\mathcal{M}$ are called events in physics and g is referred to as a Lorentz scalar product on $\mathcal{M}$.

An event $x \in \mathcal{M}$ expressed in the orthonormal basis $\{e_1, e_2, e_3, e_4\} = \{e_a\}$ of $\mathcal{M}$ is written as $x = x^a e_a = x^1 e_1 + x^2 e_2 + x^3 e_3 + x^4 e_4$. Here (x_1, x_2, x_3, x_4) are called coordinates of x relative to the basis $\{e_a\}$, with the spatial (x_1, x_2, x_3, x_4) are called coordinates of x relative to the basis $\{e_a\}$, with the spatial (x_1, x_2, x_3) and time (x_4) coordinates. Choosing two elements v, w in $\mathcal{M}$ relative to same basis of $\mathcal{M}$, one has $v = v^a e_a$, $w = w^a e_a$, and

$$g(v, w) = v^1 w^1 + v^2 w^2 + v^3 w^3 - v^4 w^4.$$

Lorentz Group

Definition 1.23 (Linear and orthogonal transformations). *If $\{e_a\}$ and $\{\hat{e}_a\}$ are two orthonormal bases for $\mathcal{M}$ then there is a unique linear transformation $L: \mathcal{M} \to \mathcal{M}$ such that $L(e_a) = \hat{e}_a$ for each $a = 1,2,3,4$. The linear transformation $L: \mathcal{M} \to \mathcal{M}$ is said to be an orthogonal transformation of $\mathcal{M}$ if $g(Lx, Ly) = g(x, y)$ for $\forall x, y \in \mathcal{M}$, so, it preserves the length of vectors.*

LORENTZ TRANSFORMATIONS

The Lorentz transformation from a moving frame (x', ct') to a frame (x, ct) at rest is given by

$$x = \gamma\left(x' + \frac{v}{c} ct'\right)$$
$$ct = \gamma\left(ct' + \frac{v}{c} x'\right)$$

We can simplify things still further. Introduce the *rapidity* β via

$$\frac{v}{c} = \tanh\beta$$

Inserting this into the expression for γ we obtain

$$\gamma = \frac{1}{\sqrt{1 - \tanh^2\beta}} = \sqrt{\frac{\cosh^2\beta}{\cosh^2\beta - \sinh^2\beta}} = \cosh\beta$$

and

$$\frac{v}{c}\gamma = \tanh\beta\cosh\beta = \sinh\beta$$

Inserting these identities into the Lorentz transformations above brings them to the remarkably simple form

$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\beta & \sinh\beta \\ \sinh\beta & \cosh\beta \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Lorentz transformations are just hyperbolic rotations!

Mathematical Properties of Lorentz Transformation

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

- l' Length measured from an observer outside the frame of reference.
- l Length measured from an observer inside the frame of reference.
- v Speed of the object.
- c Speed of light

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- t' Time measured from an observer outside the frame of reference.
- t Time measured from an observer inside the frame of reference.
- v Speed of the object.
- c Speed of light

Institute of Mathematical Sciences, Faculty of Science

Machine Learning: Application of Random Forest Algorithms in Developing Regression Model from Lamb Carcass C-Site Fat Depth Data and Comparison with Multiple Linear Regression

Sin Jie

Supervisor: Dr. Elayaraja Aruchunan & Dr. Nur Anisah Binti Mohamed @ A. Rahman

Abstract - This study proposes Random Forest Regression and Multiple Linear Regression to predict the lamb carcass C-site fat. Result shows that Multiple Linear Regression with K-means Clustering has the best performance in terms of the accuracy measurements such as mean square error (MSE), root mean square error (RMSE), R-squared (R^2), adjusted R-squared (adjusted R^2) and mean absolute error (MAE). However, as Multiple Linear Regression with K-means Clustering did not meet the assumption of multicollinearity, thus, the second-best performed method, Random Forest Regression with K-means Clustering become the best model to predict the fat depth of lamb carcass at C-site in this study.

Objective -
1. To build a suitable model to predict the fat depth of lamb carcass at C-site.
2. To compare the predictive accuracy of Random Forest Regression model and Multiple Linear Regression model in lamb carcass C-site fat depth dataset.

Introduction

As the fat depth increase, the trimming cost increase, then the market value of the lamb carcass decrease. Thus, instead of the traditional fat measurement method such as manual fingertip palpation, Murdoch University developed an energy-efficient and portable microwave device to estimate the fat depth of the lamb carcass at C-site. By using the dataset provided by Murdoch University, this study proposes Random Forest Regression and Multiple Linear Regression to predict the lamb carcass C-site fat. This study aims to build a suitable model to predict the fat depth of lamb carcass at the C-site and compare the predictive accuracy of Random Forest Regression model and Multiple Linear Regression model in lamb carcass C-site fat depth dataset.

Figure 1 Microwave device developed by Murdoch University

Methodology

Random Forest Regression

Random forest regression is an ensemble learning algorithm based on a large number of decision trees. To create a single decision tree, it employs bootstrap method to extract randomized samples from original samples. A random feature subspace is utilized to select a sorting point at each node of decision tree. Finally, these decision trees are integrated to get the final forecast result using a majority vote.

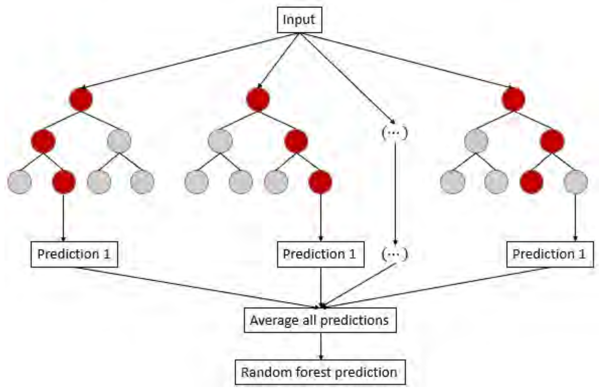


Figure 2 Illustration of Random Forest Regression

Multiple Linear Regression

Multiple linear regression is a traditional statistical method that has been widely used in identifying the linear association between the independent variables x and dependent variable y.

Principal Component Analysis (PCA)

Principal Component Analysis, or PCA, is a dimensionality-reduction method that is often used to reduce the dimensionality of large data sets, by transforming a large set of variables into a smaller one that still contains most of the information.

K-mean Clustering

K-means clustering divides the dataset into K unique clusters that do not overlap.

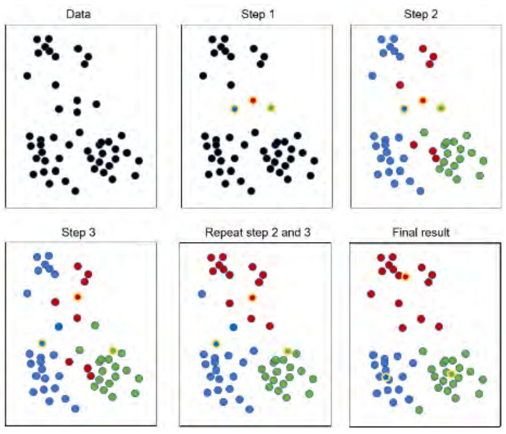


Figure 3 Illustration of K-means Clustering

Steps of K-means clustering:

1. Randomly select K points as the initial centroids.
2. Assign all points to their closest centroid in terms of Euclidean distance.
3. Recompute the centroids of each cluster.
4. Repeat steps 2 and 3 until centroids converge and remain constant.

Results and Discussion

Model	MSE	RMSE	R^2	Adjusted R^2	MAE
Random Forest Regression	1.37	1.16	0.59	1.25	0.94
Random Forest Regression with PCA	1.58	1.24	0.54	0.48	0.99
Random Forest Regression with K-means	1.33	1.13	0.60	0.58	0.89
Multiple Linear Regression	2.22	1.47	0.33	1.42	1.18
Multiple Linear Regression with PCA	1.36	1.15	0.60	0.53	0.90
Multiple Linear Regression with K-means	**1.15**	**1.06**	**0.65**	**0.64**	**0.84**

Table 1 Predictive accuracy of each model

From the table above, we observed that Random Forest Regression (RFR) and Multiple Linear Regression (MLR) without unsupervised learning have the adjusted R-squared 1.25 and 1.42 respectively, which is larger than 1. Hence, RFR model and MLR model without unsupervised learning method are unreliable. Principal Component Analysis (PCA) and K-means Clustering have been carried out later to reduce the dimensions.

MLR with K-means Clustering performs the best within these four models. This was evident across all measurements, with MSE and RMSE being 1.15 and 1.06, respectively. The lowest value of MSE and RMSE indicates that the data in Multiple Linear Regression with K-means clustering are the closest to the line of best fit. Besides, the MAE of Multiple Linear Regression with K-means Clustering is the lowest, which is 0.84. This indicates that Multiple Linear Regression with K-means Clustering has the lowest mean of residuals.

Also, MLR with K-means Clustering yield the highest R-squared and adjusted R-squared value, which are 0.65 and 0.64 respectively, this indicates that the independent variables could explain 65% of the dependent variable before adjusted and 64% of the dependent variable after adjusted. We expect that MLR with K-means Clustering is the best model, but assumption checking must be carried out before making conclusion.

Assumption Checking of Multiple Linear Regression

Cluster	VIF
0	8.54
1	22.52
2	59.82
3	44.10

Table 2 Multicollinearity Assumption Checking

The assumptions of linearity, normality, homoscedasticity, and independence are fulfilled. However, the assumption of multicollinearity does not meet. The high VIF value (larger than 10) indicates that the independent variables are correlated with each other. **As there is a violation of assumption, the Multiple Linear Regression model with K-means clustering becomes unreliable.** Hence we that the second-best performed model, Random Forest Regression with K-means Clustering is the best model to predict the fat depth of lamb carcass at C-site.

Conclusion and Future Work

Multiple Linear Regression with K-means clustering has the best performance, but it did not meet the assumption of multicollinearity; thus, the second-best performed method, **Random Forest Regression with K-means Clustering, became the best model to predict the fat depth of lamb carcass at C-site** in this study. Hence, we conclude that Random Forest Regression perform better since there are not many assumptions to be met as Multiple Linear Regression.

Due to the limited sample size of data in this study, further training and validation should be carried out to improve the model in predicting the fat depth of lamb carcass at the C-site.

Selected References

Marimuthu, J., Loudon, K. M. W., & Gardner, G. E. (2021a). Prediction of lamb carcass C-site fat depth and GR tissue depth using a non-invasive portable microwave system. *Meat Science*, *181*, 108398. https://doi.org/10.1016/j.meatsci.2020.108398

Acknowledgements

I would like to express my deepest gratitude to my project supervisors, Dr Elayaraja Aruchunan and Dr Nur Anisah for their guidance, encouragement, and inspiration.

Application of Partial Least Square Algorithm in Developing Model of Fat Depth Measurement

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SIN3015 Mathematical Science Project
Semester 1 2021/ 2022

01 INTRODUCTION

Problem Statement

Carcase trading in Australia is mostly dependent on carcase weight, with the quantity of subcutaneous fat affecting the carcase's saleable meat production. As the decreasing of the output of the lean meat, the increasing labour inputs are necessary to trim the surplus fat to achieve customer acceptability. A non-invasive and non-destructive techniques has therefore been developed which is using the Microwave System (MiS) to determine the fat in carcase.

Objective

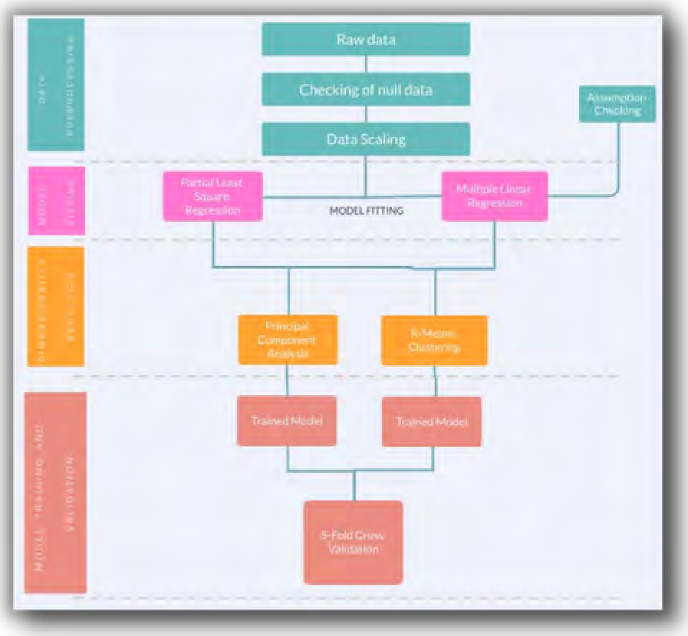
To develop a fat depth prediction model based on non-invasive techniques using a low-cost portable Microwave system.

Scope

To propose a Partial Least Square Regression model and investigate with Multiple Regression Model.

Cross validation	5-fold
Accuracy measure	R-squared, Adjusted R-squared, Root Mean Square Error, Mean Square Error, Mean Absolute Error
Environment	Jupyter Notebook
Language	Python
Code	Built-in Code and Source Code

Work's Flow Chart



02 LITERATURE REVIEW

Subjective palpation estimates or invasive objective cut techniques are the current Australian industry standards for evaluating single site fat and tissue depth in lamb carcasses (Anonymous, 2005). Microwave System (MiS) is more preferable as the frequency of microwave is able to differentiate the distinct various layers especially the biological tissues that exhibit a high difference in properties of dielectric (Marimuthu et al., 2016). MiS is a microwave measurement method which is active and based on inverse scattering (Marimuthu et al., 2016).

03 RESEARCH METHODOLOGY

Principal Component Analysis — Scatterplots are two-dimensional graphs that project multivariate data into a two-dimensional space defined by only two variables. The purpose of Principal Component Analysis is to get intrinsic variability in the data.

K-Means Clustering — K-means clustering is a type of partitional clustering where it divide data objects such that there are non-overlapping subsets divided and each of data object belongs to only one of them.

Multiple Linear Regression — One of the statistical supervised learning is the building of regression models. Often, the designing, analyzing and discovering of a model that based of a sample from a population is known as regression analysis.

Partial Least Square Regression — Partial Least Squares is a supervised way to say that the best explain of predictors in that direction is the best directions and contributing to predict the response.

R-squared — The variation proportion that is explained by the predictor variables.

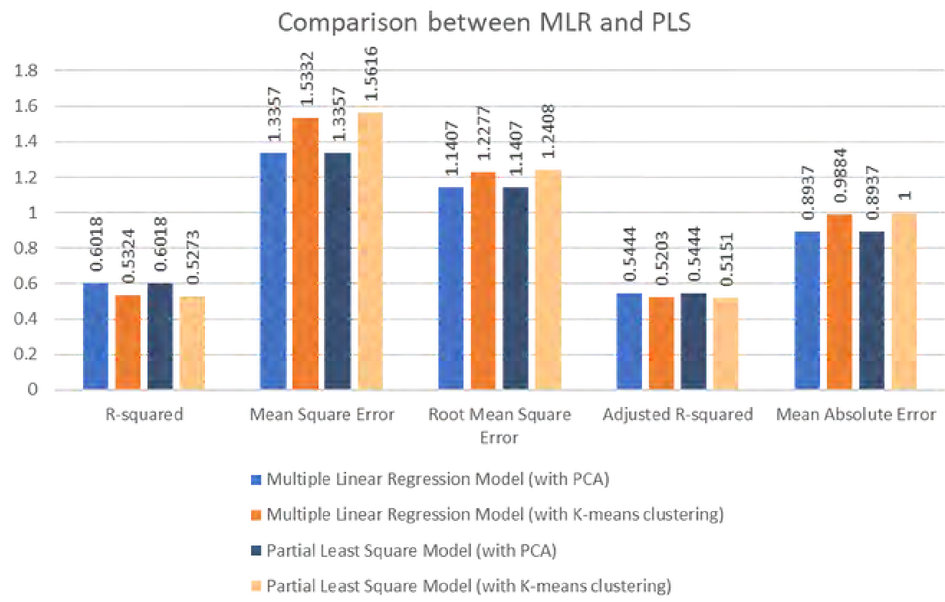
Mean Squared Error — The measure of mean error that explained the performance to predict the outcome of an observation.

Root Mean Squared Error — Average squared difference between the predicted value and the observed value.

Adjusted R-squared — Adjusts the Root Squared when there is too many variables in a model.

Mean Absolute Error — Mean absolute difference between the predicted value and the observed value.

04 RESULTS AND DISCUSSIONS



Both the Multiple Linear Regression model and Partial Least Square model with Principal Component Analysis have a similar value for all accuracy measures. Both models with K-means clustering performed slightly poorer than using Principal Component Analysis.

05 CONCLUSION

The main objective achieved which is to build a prediction model of fat depth. Since multicollinearity occurred in the data, it can be concluded that Partial Least Square is much better than the Multiple Linear Regression model in terms of R-Squared.

The future work that can be done is to further develop Multiple Linear Regression to Lasso Regression and Partial Least Square to orthogonal projection to latent structures or non-linear kernel Partial Least Square. Also, another dimension that might interested reader for a further work is that instead of using a dimension reduction method, reader may want to try other shrinkage method or variable selection method prior to model building process.

06 REFERENCES

*Note that only a few references are quoted here.

What is partial least squares regression? Retrieved from https://support.minitab.com/en-us/minitab/18/help-and-how-to/modeling-statistics/regression/supporting-topics/partial-least-squares-regression/what-is-partial-least-squares-regression/

Anonymous. (2005). Handbook of Australian Meat (7th ed.) (A. Ltd Ed. Vol. Vol. version 3).

Campbell, A., & Ntobedzi, A. (2007). Emotional intelligence, coping and psychological distress: a partial least squares approach to developing a predictive model. E-journal of applied psychology, 3, 39-54.

Mathematical Science Project

THERMAL ANALYSIS OF VLSI SYSTEM USING SOR METHOD

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Abstract

As the size of VLSI system is getting smaller nowadays, thermal profile is playing an important role in the system since the high temperature can affect the performance of the system. There is a various of method in determining the thermal profile of VLSI system. The focus of this study is to use CN method in discretization of the heat equation to obtain the linear system. The linear system will be solved by using iterative methods, GS and the proposed SOR method. Then, a comparison of the efficiency between these two methods will be analysed based on number of iterations, computational time and the maximum temperature. The numerical results will show that SOR is more efficient compare than GS. Therefore, the results for this study may be beneficial for the future research in solving numerical iterative method.

Objectives

- To discretize function of heat equation using Crank-Nicolson (CN) method and form a linear system
- To apply Successive Over Relaxation (SOR) method in solving the generated system of linear equation from the heat equation
- To analyse and compare the performances of SOR method with GS method in solving the generated system of linear equation

Heat Equation

Heat diffusion equation

$$\rho C_\rho \frac{\partial T(\vec{r},t)}{\partial t} = \nabla \cdot [\mathrm{k}(\vec{r},T)\nabla T(\vec{r},T) + g(\vec{r},T)]$$

subject to

$$k(\vec{r},T)\frac{\partial T(\vec{r},t)}{\partial n_i} + h_i T(\vec{r},T) = f_i(\vec{r},T)$$

Numerical Approach

- Crank-Nicolson (CN) method: averaging the Forward Difference Method (FDM) and Backward Difference Method (BDM) at n+1

$$T_{i,j}^{n+1} - T_{i,j}^{n} = \frac{\mathrm{k}\Delta t}{\rho C_\rho 2h^2}\left(T_{i+1,j}^{n+1} + T_{i-1,j}^{n+1} + T_{i,j+1}^{n+1} + T_{i,j-1}^{n+1} - 4T_{i,j}^{n+1} + T_{i+1,j}^{n} + T_{i-1,j}^{n} + T_{i,j+1}^{n} + T_{i,j-1}^{n} - 4T_{i,j}^{n}\right) + \frac{g_{i,j}^{n}\Delta t}{\rho C_\rho}$$

- Gauss-Seidel (GS) method: approximately like Jacobi method, but it uses updated values in each iteration

$$T_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}T_j^{(k+1)} - \sum_{j=1+1}^{n} a_{ij}T_j^{(k)}\right]$$

- Successive Over Relaxation (SOR) method: derived from GS method by applying acceleration parameter, ω

$$T_i^{(k+1)} = (1-\omega)T_i^{(k)} + \frac{1}{a_{ii}}\left[\omega\left(b_i - \sum_{j=1}^{i-1} a_{ij}T_j^{(k+1)} - \sum_{j=1+1}^{n} a_{ij}T_j^{(k)}\right)\right]$$

Result

N	Methods	Number of Iterations	CPU time (seconds)	T_{max} (K)
16 × 16	GS	353	0.32	295.708
	SOR	45 ($\omega = 1.6$)	0.16	295.708
32 × 32	GS	1266	2.08	295.7138
	SOR	114 ($\omega = 1.8$)	0.67	295.7138
64 × 64	GS	4484	28.02	295.7153
	SOR	186 ($\omega = 1.9$)	3.85	295.7153
128 × 128	GS	15624	308.32	295.7155
	SOR	798 ($\omega = 1.9$)	58.06	295.7155
256 × 256	GS	53146	4922.04	295.7148
	SOR	2792 ($\omega = 1.9$)	717.52	295.7148

Parameters	Percentage of Reduction				
	Mesh Size				
	16 × 16	32 × 32	64 × 64	128 × 128	256 × 256
Number of iterations	87.2521	90.9953	95.8519	94.8925	94.7465
Computational time(seconds)	50.0000	67.7885	86.2598	81.1689	85.4223

Conclusion

- The CN discretization method was effectively formulated and then applied to discretize the heat diffusion equation.
- By represent the system of linear equations in matrix form, we successfully implemented the proposed method and solve the problem.
- We also manage to show that SOR method is better than GS method

References

Thevaraja, M. and E. Light (2016). "Applying the Successive Over-relaxation Method to a Real World Problems." American Journal of Applied Mathematics and Statistics 4(4): 113-117.
Weste, N. H. E. and D. M. Harris (2004). CMOS VLSI Design; A Circuits and Systems Perspective, Addison Wesley.
Zhan, Y., et al. (2007). "Thermally Aware Design." Foundation and Trends in Electronic Design Automation2(3): 225-370.
Heriz, V. M., et al. (2007). Method of images for the fast calculation of temperature distributions in packaged VLSI chips. Thermal Investigation of ICs and Systems. Budapest, Hungary.
Pedram, M. and S. Nazarian (2006). "Thermal Modeling, Analysis and Management in VLSI Circuits: Principles and Methods." Proceedings of the IEEE 94(8): 1487-1501.

UNIVERSITY OF MALAYA

A STUDY ON HOMOMORPHISMS

SUPERVISOR:
PROF. DR. ANGELINA CHIN YAN MUI

INTRODUCTION

Homomorphism is a structure-preserving map $f: G \to K$ between two algebraic objects G and K, of the same type.

DEFINITION

Let $f: G \to K$ be a mapping of two algebraic objects of the same type. f is a homomorphism

1. If $\boldsymbol{G}, \boldsymbol{K}$ are **groups**,
   $\boldsymbol{f(ab) = f(a)f(b)}$ $\quad \forall a, b \in G$
2. If $\boldsymbol{G}, \boldsymbol{K}$ are **rings**,
   $\boldsymbol{f(ab) = f(a)f(b)},$
   $\boldsymbol{f(a+b) = f(a) + f(b)}$ $\quad \forall a, b \in G$
3. If **R** is a **ring with identity** and $\boldsymbol{G}, \boldsymbol{K}$ are **left $\boldsymbol{R}$-modules**,
   $\boldsymbol{f(a+b) = f(a) + f(b)},$
   $\boldsymbol{f(ra) = rf(a)}$ $\quad \forall a, b \in G, r \in R$

EXAMPLE

1. The map $f: G \to K$ that sends x to $\log_{10} x$ is a **group homomorphism**.
2. The map $f: \mathbb{Z}_2 \to \mathbb{Z}_2$ that sends x to x^2 is a **ring homomorphism**.
3. An abelian group homomorphism is also a **$\mathbb{Z}$-module homomorphism**.

GROUP ISOMORPHISM THEOREMS

The First Isomorphism Theorem

Let $\phi: G \to G'$ be an onto group homomorphism. Let $K = \ker \phi$. Then $\boldsymbol{G/K \cong G'}$

The Second Isomorphism Theorem

Let G be a group, $A \leq G$ and $N \lhd G$. Then $\boldsymbol{A/A \cap N \cong NA/N}$

Consider the groups $G = \mathbb{Z}$, $A = a\mathbb{Z}$ and $N = b\mathbb{Z}$. For any positive integers a, b,
$$\frac{a\mathbb{Z} + b\mathbb{Z}}{b\mathbb{Z}} \cong \frac{a\mathbb{Z}}{a\mathbb{Z} \cap b\mathbb{Z}}$$

The Third Isomorphism Theorem

Let G be a group with normal subgroups A and N such that $N \lhd A \lhd G$. Then $\boldsymbol{(G/N)/(A/N) \cong G/A}$

Given G is a finite group and $H, K \lhd G$ with $H \subseteq K$. Then $(G/H)/(K/H) \cong G/K$. G/K is abelian if G/H is abelian.

RINGS

MODULES

The First Isomorphism Theorem

Let $\phi: R \to S$ be an onto ring homomorphism. Then $\boldsymbol{R/\ker\phi \cong S}$

Let $f: M \to N$ be a module homomorphism with kernel K. Then $\boldsymbol{M/K \cong f(M)}$

The Second Isomorphism Theorem

Let R be a ring, $S \leq R$ and $I \lhd R$. Then $\boldsymbol{S/(S \cap I) \cong (S+I)/I}$

Let S and T be submodules of M. Then $S+T$ and $S \cap T$ are submodules of M and $\boldsymbol{S/(S \cap T) \cong (S+T)/T}$

The Third Isomorphism Theorem

Let R be a ring and let $J \subseteq I$ be ideals of R. Then $\boldsymbol{(R/J)/(I/J) \cong R/I}$

Let M be a module with submodules N, L such that $N \leq L \leq M$. Then $\boldsymbol{M/L \cong (M/N)/(L/N)}$

By the Third Ring Isomorphism Theorem, it can be verified that $(3, x^3 + 2x + 2)$ is a maximal ideal of $\mathbb{Z}[x]$.

TYPES OF HOMOMORPHISM

Monomorphism ➜ f is one-to-one

Epimorphism ➜ f is onto

Isomorphism ➜ f is one-to-one and onto $\boldsymbol{(G \cong K)}$

KERNEL OF HOMOMORPHISM

Let $f: G \to K$ be a homomorphism between the same algebraic objects. Then the kernel of f is
$\boldsymbol{\ker f = \{x \in G : f(x) = 1_K\}}$
for groups and
$\boldsymbol{\ker f = \{x \in G : f(x) = 0\}}$
for rings and modules.

CONCLUSION

- ✓ Homomorphism is an useful tool.
- ✓ The Isomorphism Theorems allow us to identify two seemingly different algebraic objects as one.

REFERENCES

Ikenaga, B. (2018b). The First Isomorphism Theorem. Retrieved January 7, 2022, from https://sites.millersville.edu/bikenaga/abstract-algebra-1/first-isomorphism-theorem/first-isomorphismtheorem.html
Scherr, Z. (2015). Math 371 - Ring Isomorphism Theorems. Retrieved January 7, 2022, from http://eg.bucknell.edu/~zls002/Math371Sp15/Notes/IsoThms.pdf
Applications of the Isomorphism theorems. (2013, February 20). Retrieved January 7, 2022, from https://math.stackexchange.com/questions/308942/applications-of-the-isomorphism-theorems

NURHIDAYAH BINTI KHAMIRIN (17121745/2) *SIN3015 MATHEMATICAL SCIENCE PROJECT*

ON THE HAMILTONICITY OF CUBIC GRAPHS

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Abstract

A hamiltonian cubic graph is a graph with vertices of degree 3 that contains a Hamilton cycle that passes through all the vertices. In this project, we discuss some conjectures circulating on the hamiltonicity of cubic 3-connected planar graphs (C3CPs) and cubic 3-connected bipartite graphs (C3CBs).

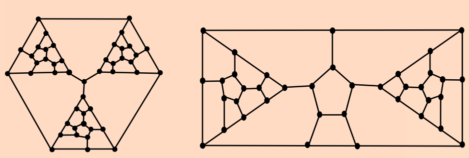
We also present some C3CPs, C3CBs and k-pieces that possess special properties. By using some of these C3CPs and k-pieces, we able to construct some non-hamiltonian C3CPs.

Objectives

- To discuss the **hamiltonicity of cubic graphs** focusing on planar and bipartite graphs
- To discuss on some **conjectures** related to hamiltonicity of cubic graphs
- To **construct some non-hamiltonian cubic 3-connected graphs** using some C3CPs and 3-pieces with special properties

Tait's Conjecture (1880)

Every C3CP graph is hamiltonian.



Left to right: The first counterexample of 46 vertices due to Tutte (1946) and of the 6 non-isomorphic non-hamiltonian C3CPs due to Holton and McKay (1988).

Theorem 3.1 (Holton and McKay, 1988) *Every C3CP with 36 or fewer vertices is hamiltonian.*

Barnette's Conjecture (1969)

Remains Open

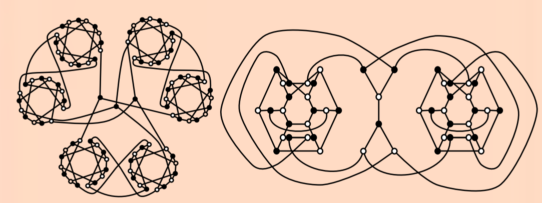
Every C3CBP graph is hamiltonian.

One of the partial results:

Theorem 3.1 (Holton and McKay, 1985) *Let G be a C3CBP on n vertices. If $n \leq 64$, then G is hamiltonian.*

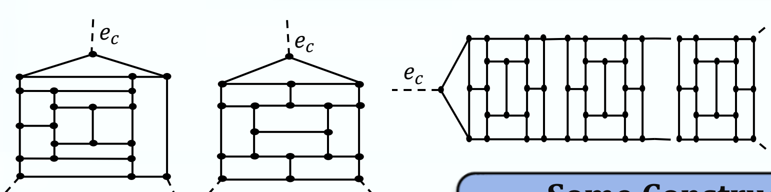
Tutte's Conjecture (1971)

Every C3CB graph is hamiltonian.

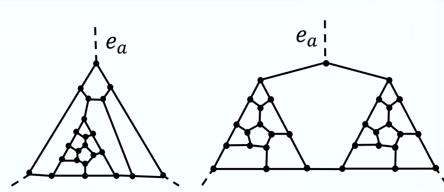


Left to right: The first counterexample of 96 vertices due to Horton (1976) and the smallest non-hamiltonian C3CB due to Georges (1989).

Some 3-pieces with Compulsory Edges of Attachment

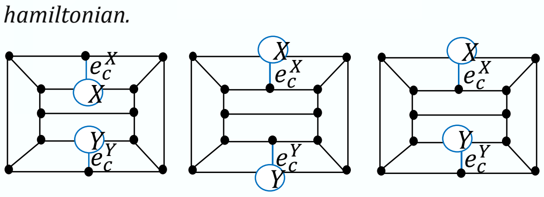


3-pieces with Compulsory Edges of Attachment

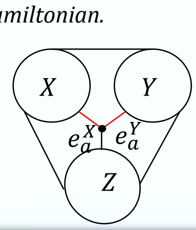


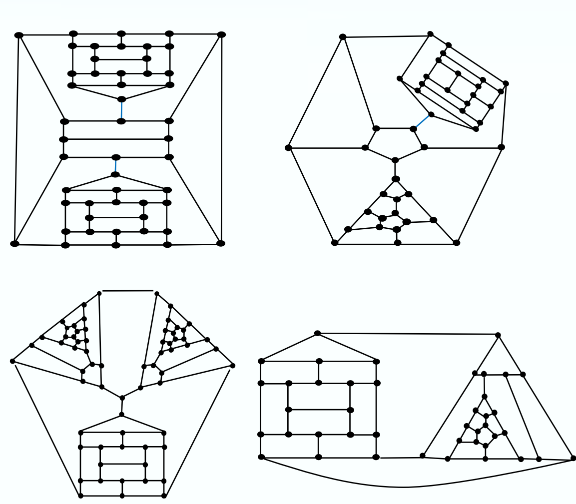
Some Construction of Non-hamiltonian C3CPs (Some Project's Results)

Theorem 1 *Let X and Y be the subgraph of a C3CP graph G as shown below. Let e_c^X and e_c^Y be the compulsory edge of X and Y, respectively. Then G is non-hamiltonian.*

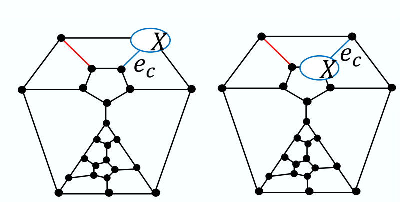


Theorem 3 *Let X, Y and Z be subgraphs of a C3CP graph G as shown below. If e_a^X, and e_a^Y are the impossible edges of attachment of X, and Y, respectively, then G is non-hamiltonian.*

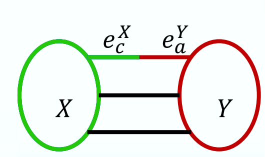




Theorem 2 *Let X be the subgraph of a C3CP graph G as shown below. Let e_c be the compulsory edge of X. Then G is non-hamiltonian.*



Theorem 4 *Let X and Y be subgraphs of a C3CP graph G as shown below. If e_c^X the impossible edge of attachment of X, and e_a^Y the compulsory edge of attachment of Y, then G is non-hamiltonian.*



References

Georges, J. P. (1989). Non-hamiltonian bicubic graphs. *Journal of Combinatorial Theory, Series B*, *46*(1), 121–124. https://doi.org/10.1016/0095-8956(89)90012-9

Holton, D., Manvel, B., & McKay, B. (1985). Hamiltonian cycles in cubic 3-connected bipartite planar graphs. *Journal of Combinatorial Theory, Series B*, *38*(3), 279–297. https://doi.org/10.1016/0095-8956(85)90072-3

Holton, D., & McKay, B. (1988). The smallest non-hamiltonian 3-connected cubic planar graphs have 38 vertices. *Journal of Combinatorial Theory, Series B*, *45*(3), 305–319. https://doi.org/10.1016/0095-8956(88)90075-5

Tutte, W. (1971). On the 2-factors of bicubic graphs. *Discrete Mathematics*, *1*(2), 203–208. https://doi.org/10.1016/0012-365x(71)90027-6

GENERALISED KNIGHT'S TOUR PROBLEM ON THE RECTANGULAR BOXES

SIN3015 MATHEMATICAL SCIENCE PROJECT

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INTRODUCTION

A knight's tour is a sequence of moves of a knight on a chessboard such that the knight visits every square exactly once. An m x n chessboard is an array with m rows and n columns of an arranged square cells. An (a, b)-knight's move is the result of moving a cells horizontally or vertically and then moving b cell perpendicularly to the direction. If a knight returns to its starting cell, it is called closed knight's tour; otherwise it is called open knight's tour. The knight's tour problem is corresponding to determine whether a knight graph is Hamiltonian in graph theory.

OBJECTIVES

- •Discuss the knight's tour problem on rectangular chessboards and some surfaces.
- •Discuss the generalized (a, b) – knight's tour problem on rectangular chessboards.
- •Discuss the generalized (a, b) – knight's tour problem on cylinders.
- •Obtain some results for the (2, 3)–knight's tour problem within m x n x k boxes.

(1, 2)-KNIGHT'S TOUR ON RECTANGULAR CHESSBOARDS

- Closed (1, 2)-knight's tours

Theorem 1 (Schwenk,1991) : An m x n chessboard with $m \le n$ admits a closed knight's tour unless one or more of these three condition holds:
(a) m and n are both odd; (b) m = 1, 2, or 4; (c) m = 3 and n = 4, 6, or 8.

- Open knight's tours

Theorem 2 (Cull and de Curtins,1978) : Every m x n chessboard with $5 \le m \le n$ admits an open knight's tour

Theorem 3 (Chia and Ong, 2005) : The m x n chessboard with $m \le n$ admits an open knight's tour unless one or more of the following conditions holds:
(i)m = 1 or 2; (ii)m = 3 and n = 3, 5, or 6; (iii) m = 4 and n = 4

(1,2)-KNIGHT'S TOUR ON RECTANGULAR CHESSBOARDS ON SURFACES

- Knight's tour problems on cylinders

Theorem 4 (Watkins, 2000) : On cylinder, the knight admits a closed knight's tour on the m x n chessboards unless one of the two conditions holds:
(i)m =1 and n > 1; (ii)m = 2 or 4 and n is even.

- Knight's tour problems on torus

Theorem 5 (Watkins, 1997) : On torus, assume $m \le n$, the knight admits a closed knight's tour on every m x n chessboard

- Knight's tour problems on boxes

Theorem 6 (Qing and Watkins, 2006) : Every m x n box admits a closed knight's tour on their surface

Theorem 7: (Qing and Watkins, 2006) : Every 4m x 4n x 4k solid box admits a closed knight's tour

(a, b)-KNIGHT'S TOURS ON RECTANGULAR CHESSBOARDS

Theorem 8 (Chia and Ong, 2005) : Suppose the m x n chessboard admit a closed (a, b)-knight's tour, where a < b and $m \le n$. Then
(i)a + b is odd; (iii)$m \ge a + b$; and
(ii)m or n is even; (iv)$n \ge 2b$.

(2, 3)-KNIGHT'S TOUR ON RECTANGULAR CHESSBOARDS

Theorem 9: [Chia and Ong, 2005] There is no closed (2, 3) – knight's tour on the 7 x n chessboard.
(ii) If $m \le 4$ or m = 6, 7, 8, 12, then the m x n chessboard does not admit a closed (2, 3) – knight's tour.
(iii) Suppose $n \ne 18$. Then the 5 x n chessboard admits a closed (2, 3) – knight's tour if and only if $n \ge 16$ is even.

(2, 3) - KNIGHT'S TOUR ON CYLINDERS

- Theorem 10 (Sirirat et al, 2019) : On cylinder, the knight admits no closed (2, 3)-knight's tours on the m x n chessboard when $m \le 8$ and $m \ne 5$ for any positive integer n.
- Theorem 11 (Sirirat et al, 2019) : On cylinder, the knight admits no closed (2, 3)-knight's tours on the 5k x n chessboard for any positive integer k and n.

PROJECT'S PROBLEM

Which chessboards admit an open or a closed (a, b)-knight's tour within m x n x k boxes?

- Closed (2, 3) - knight's tours

Theorem 12 : Let a < b and $m \le n$. If
(i) a + b is even; (iii) m, k < a + b; or
(ii) m, n and k are all odd; (iv) n < 2b, then
the m x n x k chessboard on a solid box does not admit closed (a, b)-knight's tour within the m x n x k boxes.

Corollary 13: If m, $k \le 4$ or n < 6, then the knight admits no closed (2, 3) – knight's tour within the m x n x k boxes.

Theorem 14: For n = 16, 20, 24 and $k \ge 6$, the knight admits a closed (2, 3)-knight's tour within the 5 x n x k boxes

- Open (2, 3) - knight's tour

Theorem 15: For n = 16, 20, 22, 24 and $k \ge 5$, the knight admits an open (2, 3)-knight's tour within the 5 x n x k boxes.

In this project , we will focus on the (2, 3) – knight's tour within 5 x n x k boxes.

425	412	407	402	469	376	391	22	419	366	465	362	381	396	463	370	455	444	449	386
414	405	24	409	400	423	472	379	398	421	364	393	460	373	388	453	446	383	458	451
403	470	377	26	411	416	367	468	375	390	395	418	369	466	443	448	385	462	371	456
408	401	424	413	406	23	420	365	392	473	380	397	464	363	382	459	450	387	454	445
25	410	415	404	471	378	399	422	417	368	467	374	389	394	461	372	457	452	447	384

172	185	190	195	162	221	206	167	178	231	258	235	216	201	256	227	248	237	242	211
183	192	169	188	197	174	165	218	199	176	233	204	253	224	209	246	239	214	251	244
194	163	220	171	186	181	230	261	222	207	202	179	228	259	236	241	212	255	226	249
189	196	173	184	191	168	177	232	205	166	217	200	257	234	215	252	243	210	247	238
170	187	182	193	164	219	198	175	180	229	260	223	208	203	254	225	250	245	240	213

7	436	431	426	17	36	51	12	1	500	21	496	41	56	475	30	483	494	489	46
438	429	10	433	60	5	14	39	58	3	498	53	478	33	48	485	492	43	480	487
427	16	37	8	435	440	27	18	35	50	55	442	29	20	495	490	45	476	31	482
432	61	6	437	430	11	2	499	52	13	40	57	474	497	42	479	488	47	484	493
9	434	439	428	15	38	59	4	441	28	19	34	49	54	477	32	481	486	491	44

332	345	350	355	322	281	266	327	338	291	318	295	276	361	316	287	308	297	302	271
343	352	329	348	357	334	325	278	359	336	293	264	313	284	269	306	299	274	311	304
354	323	280	331	346	341	290	321	282	267	262	339	288	319	296	301	272	315	286	309
349	356	333	344	351	328	337	292	265	326	277	360	317	294	275	312	303	270	307	298
330	347	342	353	324	279	358	335	340	289	320	283	268	263	314	285	310	305	300	273

143	156	161	66	133	92	77	138	149	102	129	106	87	72	127	98	119	108	113	82
154	63	140	159	68	145	136	89	70	147	104	75	124	95	80	117	110	85	122	115
65	134	91	142	157	152	101	132	93	78	73	150	99	130	107	112	83	126	97	120
160	67	144	155	62	139	148	103	76	137	88	71	128	105	86	123	114	81	118	109
141	158	153	64	135	90	69	146	151	100	131	94	79	74	125	96	121	116	111	84

Figure 1: An open (2, 3)-knight's tour within a 5 x 20 x 5 box

404	425	420	365	380	407	370	107	418	101	376	411	398	393	388	413
427	368	109	422	363	402	105	98	373	382	103	400	391	416	395	386
366	379	96	111	424	419	100	377	408	371	384	389	414	375	410	397
421	364	403	426	369	108	381	102	401	106	417	394	387	412	399	392
110	423	428	367	378	97	372	383	104	99	374	409	396	385	390	415

225	260	255	266	281	242	271	230	253	236	277	246	219	214	209	248
262	269	228	257	264	223	232	239	274	203	234	221	212	251	216	207
267	280	241	226	259	254	237	278	243	272	205	210	249	276	245	218
256	265	224	261	270	229	282	235	222	231	252	215	208	247	220	213
227	258	263	268	279	240	273	204	233	238	275	244	217	206	211	250

444	119	406	113	468	83	478	439	94	433	472	87	450	455	460	89
117	480	441	122	115	446	437	430	475	466	435	448	457	92	453	462
112	469	82	443	120	95	432	471	84	477	464	459	90	473	86	451
405	114	445	118	479	440	467	434	447	438	93	454	461	88	449	456
442	121	116	1	470	429	476	465	436	431	474	85	452	463	458	91

289	334	339	328	313	352	323	284	341	358	317	348	295	300	305	346
332	325	286	337	330	291	362	355	320	311	360	293	302	343	298	307
327	314	353	288	335	340	357	316	351	322	309	304	345	318	349	296
338	329	290	333	324	285	312	359	292	283	342	299	306	347	294	301
287	336	331	326	315	354	321	310	361	356	319	350	297	308	303	344

196	151	146	157	172	133	162	201	144	127	168	137	190	185	180	139
153	160	199	148	155	194	123	130	165	174	125	192	183	142	187	178
158	171	132	197	150	145	128	169	134	163	176	181	140	167	136	189
147	156	195	152	161	200	173	126	193	202	143	186	179	138	191	184
198	149	154	159	170	131	164	175	124	129	166	135	188	177	182	141

35	80	5	74	59	18	69	30	7	24	63	14	41	46	51	12
78	71	32	3	76	37	28	21	66	57	26	39	48	9	44	53
73	60	19	34	81	6	23	62	17	68	55	50	11	64	15	42
4	75	36	79	70	31	58	25	38	29	8	45	52	13	40	47
33	2	77	72	61	20	67	56	27	22	65	16	43	54	49	10

Figure 1: A Closed (2, 3)-knight's tour wihtin a 5 x 16 x 6 box

REFERENCES

[1] A.J. Schwenk, Which rectangular chessboards have a knight's tour?, Mathematics Magazine. 64 (1991) 325–332. [2] G.L. Chia, Siew-Hui Ong, Generalized knight's tours on rectangular chessboards, Discrete Appl. Math. 159 (2005) 80–98. [3] J.J. Watkins, Knight's tours on cylinders and other surfaces, Congressus Numerantium. 143 (2000) 117–127. [4] J.J. Watkins, R.L. Hoenigman, Knight's tours on a torus, Mathematics Magazine. 70 (1997) 175–184. [5] Y. Qing and J.J. Watkins, *Knight's Tours for Cubes and Boxes*, Congressus Numerantium 181 (2006) 41–48 [6] S.Singhun, P. Ruengmanee and A. Sinna, *A closed (2, 3)-knight's tour on some cylinder chessboards*, AKCE International Journal of Graphs and Combinatorics [7] P. Cull, De Curtins, Knight's tour revisited, Fibonacci Quart. 16 (1978) 276–285.

CURVES IN MINKOWSKI SPACE

SIN3015

FINAL YEAR PROJECT

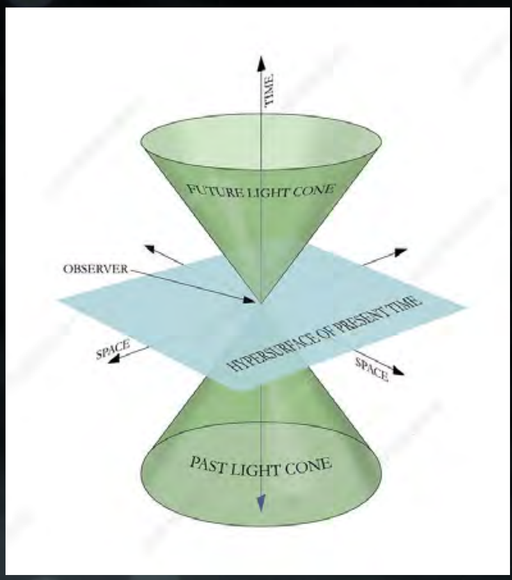
NUR NAJIA BINTI AHMAD ZAIDI

17133722/2

SUPERVISOR: DR. LOO TEE HOW

WHAT IS A MINKOWSKI SPACE

- A Minkowski space is a 4-dimensional space (a combination of 3-dimensional Euclidean space and time)
- Closely related to Einstein's theories of special relativity and general relativity
- Equipped with Minkowski metric (indefinite, non-degenerate, symmetric bilinear form)
- Minkowski metric: $g(x,y) = -x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4,\ \forall x,y \in \mathbb{E}_1^4$



Minkowski spacetime light cone diagram

VECTORS IN MINKOWSKI SPACE

Causal character of vectors in Minkowski space:

- A vector v in $\mathbb{E}_1^4$ can have one of the three causal characters:
  - spacelike if $g(v,v) > 0$ or $v = 0$
  - timelike if $g(v,v) < 0$
  - lightlike (null) if $g(v,v) = 0$ and $v \neq 0$
- The norm of a vector: $||v|| = \sqrt{|g(v,v)|}$
- Vectors v and w are orthogonal if $g(v,w) = 0$

CURVES IN MINKOWSKI SPACE

- A general helix – A curve with tangent that makes a constant angle with a fixed direction.
- A slant helix – a helix which its principal normal makes a constant angle with a fixed line in space
- The causal character of a curve $\gamma(s)$ in $\mathbb{E}_1^4$ can locally be *spacelike, timelike* or *lightlike (null)* if all its velocity vectors $\gamma'(s)$ are *spacelike, timelike* or *null* respectively

CARTAN EQUATIONS

$$\begin{bmatrix} T' \\ N' \\ B_1' \\ B_2' \end{bmatrix} = \begin{bmatrix} 0 & \kappa_1 & 0 & 0 \\ \kappa_2 & 0 & -\kappa_1 & 0 \\ 0 & -\kappa_2 & 0 & \kappa_3 \\ -\kappa_3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B_1 \\ B_2 \end{bmatrix},$$

κ_1, κ_2 *and* κ_3 are the first, second and the third Cartan curvature of γ.

k-TYPE NULL SLANT HELICES

A null Cartan curve γ with the Cartan frame $\{T, N, B_1, B_2\}$ in $\mathbb{E}_1^4$ is called a k-type null slant helix if there exists a non-zero fixed direction $U \in \mathbb{E}_1^4$ such that there holds $g(V_{k+1}, U) = const$, for $0 \leq k \leq 3$. The fixed direction U is called the axis of the helix.

0-type null slant helices are generalized null helices and 1-type null slant helices are null slant helices.

THERMAL ANALYSIS OF VLSI SYSTEM USING GEOMETRIC MEAN METHOD

UNIVERSITY OF MALAYA

Institute of Mathematical Sciences

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Introduction

- **VLSI (Very Large Scale Integration)** system is a process of fabricating more than 1000 transistors onto a chip to create an integrated circuit (IC).
- Smaller and faster devices are the high demands along with the advancement of electronic technologies.
- With reduction in feature of size, integrated circuit becoming more compact. Thus, there is no enough space to add a bigger fan to fix the thermal issue.
- This leads to the **rise of temperature** in the system.
- **Thermal management** is very important because high temperature produced can shorten interconnect and lifetime of devices.

Objectives

- To determine the thermal profile of VLSI system by implementing Crank Nicolson method to discretize 2D heat partial differential equation.
- To obtain the result by Geometric Mean iterative method.
- To compare the performance of results obtained from Gauss Seidel and Geometry Mean methods.

Scope of Study

- 2D model length is set to 4.75m
- The value of parameters are based on property of Silicon
- Boundary condition is assigned to zero
- The simulation is done using different grid size which are 16x16, 32x32, 64x64, 128x128 and 256x256

Methodology

1) Derivation of model by heat equation

Heat Equation

$$\frac{\delta T(x,y,t)}{\delta t} = \frac{k}{\rho C_p}\frac{\delta^2 T(x,y,t)}{\delta x^2} + \frac{k}{\rho C_p}\frac{\delta^2 T(x,y,t)}{\delta y^2} + \frac{1}{\rho C_p} g(x,y,t)$$

$$0 \le x, y \le L \quad t \ge 0$$

subject to

$$k(\vec{r},T)\frac{dT(\vec{r},t)}{dn_i} + h_i T(\vec{r},T) = 0$$

2) Crank Nicolson Scheme

First, replaced left side of heat equation by applying the forward difference with time.

$$\frac{\delta T(x,y,t)}{\delta t} = \frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t}$$

For the right side, Crank Nicolson is obtained by taking the average of forward and backward difference method. Thus, the second-order derivative of T with respect to x and y can be written as:

i) forward difference method

$$\left(\frac{T_{i-1,j}^{n+1} - 2T_{i,j}^{n+1} + T_{i+1,j}^{n+1}}{\Delta x^2} + \frac{T_{i,j-1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j+1}^{n+1}}{\Delta y^2}\right)$$

ii) backward difference method

$$\left(\frac{T_{i-1,j}^{n} - 2T_{i,j}^{n} + T_{i+1,j}^{n}}{\Delta x^2} + \frac{T_{i,j-1}^{n} - 2T_{i,j}^{n} + T_{i,j+1}^{n}}{\Delta y^2}\right)$$

After substituting into the heat equation, this will be reduced to:

$$(1+4\lambda)T_{i,j}^{n+1} - \lambda(T_{i-1,j}^{n+1} + T_{i,j-1}^{n+1} + \lambda T_{i+1,j}^{n+1} + \lambda T_{i,j+1}^{n+1}) = (1+4\lambda)T_{i,j}^{n} + \lambda(T_{i-1,j}^{n} + T_{i,j-1}^{n} + T_{i+1,j}^{n} + T_{i,j+1}^{n})$$

Where the parameter,

$$\lambda = \frac{k\Delta t}{2\rho C_p h^2}$$

In this study, the boundary values are assigned to 0. Right hand side is the initial temperature and the value is set under room temperature as 293.15K. Since right hand side is known, boundary values from the left side will be shifted to the right. As a result, this system can be expressed in matrix equation,

$$A = \begin{bmatrix} 1+4\lambda & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1+4\lambda & \ddots & 0 & \ddots & 0 & \vdots \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & 1+4\lambda & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & 0 & 1+4\lambda & \ddots & 0 \\ \vdots & 0 & \cdots & 0 & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & 1+4\lambda \end{bmatrix} \quad X = \begin{bmatrix} T_{1,1}^{n+1} \\ T_{2,2}^{n+1} \\ T_{3,3}^{n+1} \\ \vdots \\ \vdots \\ \vdots \\ T_{N-1,N-1}^{n+1} \end{bmatrix}$$

$$B = \begin{bmatrix} (1-4\lambda)T_{1,1}^{n} + \lambda(T_{2,1}^{n} + T_{0,1}^{n} + T_{1,0}^{n} + T_{1,2}^{n}) + \lambda T_{1,2}^{n+1} \\ (1-4\lambda)T_{2,2}^{n} + \lambda(T_{3,2}^{n} + T_{1,2}^{n} + T_{2,1}^{n} + T_{2,3}^{n}) + \lambda T_{2,1}^{n+1} + \lambda T_{2,3}^{n+1} \\ (1-4\lambda)T_{3,3}^{n} + \lambda(T_{4,3}^{n} + T_{2,3}^{n} + T_{3,2}^{n} + T_{3,4}^{n}) + \lambda T_{3,2}^{n+1} + \lambda T_{3,4}^{n+1} \\ \vdots \\ \vdots \\ \vdots \\ (1-4\lambda)T_{N-1,N-1}^{n} + \lambda(T_{N,N-1}^{n} + T_{N-2,N-1}^{n} + T_{N-1,N-2}^{n} + T_{N-1,N}^{n}) + \lambda T_{N-2,N-1}^{n+1} \end{bmatrix}$$

3) Geometric Mean Method

Formulation for Geometric Mean method , we let the matrix A splitting as stated below,

The linear system in the matrix representation will be solved by the geometry mean iterative method

$$A = D - L - U$$

where these matrices are diagonal, strictly upper triangular, and strictly lower triangular accordingly.

The general formulation for geometric mean method are defined as follows:

$$\begin{cases} (D - \omega L)x^1 = [(1-\omega)D + \omega U]x^k + \omega B \\ (D - \omega U)x^2 = [(1-\omega)D + \omega L]x^k + \omega B \\ x^{(k+1)} = (x^1 \circ x^2)^{\frac{1}{2}} \end{cases}$$

where ω, $\circ$ *and* $(.)^{\frac{1}{2}}$ denote the acceleration parameter $(0 < \omega < 2)$, Hadamard product Hadamard power accordingly.

Result & Discussion

Mesh size	Methods	Number of Iterations (k)	CPU time (t)	Tmax
16x16	GS	353	0.32	295.7080
	GM	30 (w=1.7)	0.09	295.7080
32x32	GS	1266	2.08	295.7138
	GM	56 (w=1.8)	0.36	295.7138
64x64	GS	4484	28.02	295.7153
	GM	74 (w=1.9)	1.89	295.7153
128x128	GS	15624	308.32	295.7155
	GM	112 (w=1.9)	2.37	295.7155
256x256	GS	53146	4922.04	295.7148
	GM	488 (w=1.9)	74.04	295.7148

- Geometric Mean method reduced the number of iterations and computational time significantly compared to the Gauss Seidel method.
- The larger the mesh size, the higher the number of iterations, computational time and maximum temperature.
- in terms of maximum temperature, the temperature obtained by using Geometric Mean method is comparable with the results generated via Gauss Seidel method.

Conclusion

The objectives are successfully achieved. The achievements of this project are as follows:

- The 2D heat equation is successfully discretized by using CN scheme. The implementation of CN scheme resulted in a system of linear equations (matrix form).
- GM iterative methods are managed to solve on the generated linear systems.
- Based on the Table, the performance of GM iterative method is better compared to GS

References

1) Mohana Sundaram Muthuvulu, J. S. (2011). Half-Sweep Geometric Mean Iterative Method for the Repeated Simpson Solution of Second Kind Linear Fredholm Integral Equations. Proyecciones Journal of Mathematics, 65-79.

2) D.M. Causon, C. M. (2010). Introductory Finite Difference Methods for PDEs. United Kingdom.

3)Buyan, M. H. (2017). History and Evolution of CMOS Technology and its Application in Semiconductor Industry. SEU Journal of Science and Engineering, 28-42.

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Cayley Tables and Sudoku

We study the connections between Cayley tables of finite group and Sudoku. Besides that, we investigate on three methods to construct Sudoku table using group cosets.

Let G be a group and H a subgroup of G

Construction 1

Left coset reprentatives

Right cosets

	9	3	6	1	4	7	2	5	8
9	9	3	6	1	4	7	2	5	8
1	1	4	7	2	5	8	3	6	9
2	2	5	8	3	6	9	4	7	1
3	3	6	9	4	7	1	5	8	2
4	4	7	1	5	8	2	6	9	3
5	5	8	2	6	9	3	7	1	4
6	6	9	3	7	1	4	8	2	5
7	7	1	4	8	2	5	9	3	6
8	8	2	5	9	3	6	1	4	7

Find right cosets of H and the complete set of left coset representatives (c.s.l.c.r), of H in G

Example:

G= {1,2,3,4,5,6,7,8,9} and H= {3,6,9}.

Right cosets: {3,6,9}, {4,7,1} and {5,8,2}.

c.s.l.c.r: {3,4,5}, {6,7,8} and {9,1,2}

Construction 2

- Find left cosets of H and its conjugates in G.
- Find the left cosets of conjugates and c.s.l.c.r.
- Label rows with c.s.l.c.r and columns with left cosets of H.

Example:

S_3= {1, (23), (12), (123), (132), (13)} and H = {1, (12)}.

Left cosets of H: {1, (12)}, {(13), (123)} and {(23), (132)}

Conjugates:

- H^{1}= {1, (12)} = $H^{(12)}$
- $H^{(23)}$= {(1, (13)} = $H^{(123)}$
- $H^{(13)}$= {(1, (23)} = $H^{(132)}$

Left cosets of H^{g}: {1, (12)}, {(13), (123)} and {(23), (132)}

Left coset representatives of H^{g}: {1, (13), (23)} and {(12), (123), (132)}

Left cosets

	1	(1 2)	(1 3)	(1 2 3)	(2 3)	(1 3 2)
1	1	(1 2)	(1 3)	(1 2 3)	(2 3)	(1 3 2)
(1 3)	(1 3)	(1 2 3)	1	(1 2)	(1 3 2)	(2 3)
(2 3)	(2 3)	(1 3 2)	(1 2 3)	(1 3)	1	(1 2)
(1 2)	(1 2)	1	(1 3 2)	(2 3)	(1 2 3)	(1 3)
(1 2 3)	(1 2 3)	(1 3)	(2 3)	(1 3 2)	(1 2)	1
(1 3 2)	(1 3 2)	(2 3)	(1 2)	1	(1 3)	(1 2 3)

Left coset reprentatives of H^{g}

Construction 3

Cayley-Sudoku Table of H

+	0	4	2	6
0	0	4	2	6
2	2	6	4	0
4	4	0	6	2
6	6	2	0	4

Cayley-Sudoku Table of Z_8

		$\{0,4\}+0$		$\{2,6\}+0$		$\{0,4\}+1$		$\{2,6\}+1$	
	+	0	4	2	6	1	5	3	7
$0+\{0,2\}$	0	0	4	2	6	1	5	3	7
	2	2	6	4	0	3	7	5	1
$1+\{0,2\}$	1	1	5	3	7	2	6	4	0
	3	3	7	3	1	4	0	6	2
$0+\{4,6\}$	4	4	0	6	2	5	1	7	3
	6	6	2	0	4	7	3	1	5
$1+\{4,6\}$	5	5	1	7	3	6	2	0	4
	7	7	3	1	5	0	4	2	6

- Apply Construction 1 on H to form a Cayley-Sudoku table of H.
- Let {a_1, a_2, a_3, a_4} and {b_1, b_2, b_3, b_4} be c.s.l.c.r and complete set of right coset representative of H respectively.
- Multiply columns and rows from Cayley-Sudoku table of H with {b_1, b_2, b_3, b_4} and {a_1, a_2, a_3, a_4} respectively.

Example:

Z_8= {0,1,2,3,4,5,6,7} and H = {0,2,4,6}.

Step 1

- Apply Construction 1 on H and A = {0,4} is a subgroup of H,
- Right cosets of A: {0,4} and {2,6}.
- c.s.l.c.r: {0,2} and {4,6}.

Step 2

- Right cosets of H: {0,2,4,6} and {1,3,5,7}
- Accordingly, {0,1} is the left coset and right coset representatives.
- Add columns and rows from the Cayley-Sudoku table of H with {0,1} and to get the Cayley-Sudoku table of the whole group.

UNIVERSITI MALAYA

Analysis of Covid 19 in Malaysia using SIR model

SIN3015 FINAL YEAR PROJECT
BY NIK AMMAR (17151220)
SUPERVISOR : PROF MADYA DR ZAILAN BIN SIRI

ABSTRACT

COVID-19 offers an urgent global concern because of its contagious nature, regularly changing traits, and the lack of a vaccine or effective medications. A strategy for measuring and limiting the continuing development of COVID-19 is urgently required to deliver smart health care services. Artificial intelligence, machine learning, deep learning, cognitive computing, cloud computing, fog computing, and edge computing are all examples of advanced intelligent computing. For smart health care and the well-being of Malaysia people, this research provides a methodology for predicting COVID-19 utilising the SIR model using MATLAB. For mathematical modelling to be able to determine the behavioural effects of the pandemic, knowing the number of susceptible, infected, and recovered patients each day is crucial. It predicts the situation over the next 646 days. COVID-19 will expand throughout the population or die off in the long run, according to the proposed system.

OBJECTIVE

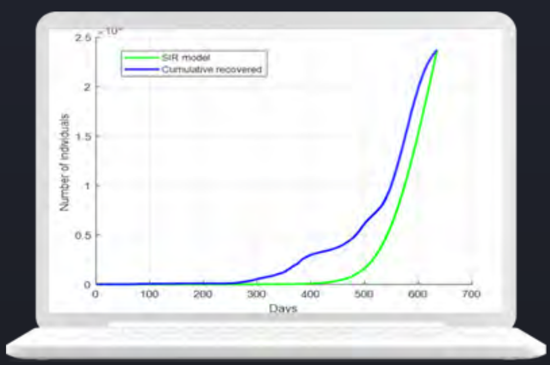
- To collect the daily covid 19 data from the reliable sources
- To use SIR model to develop approximate data to compare with actual data
- To develop MATLAB code for SIR model
- To analyse trend of covid-19 in Malaysia

SIR MODEL

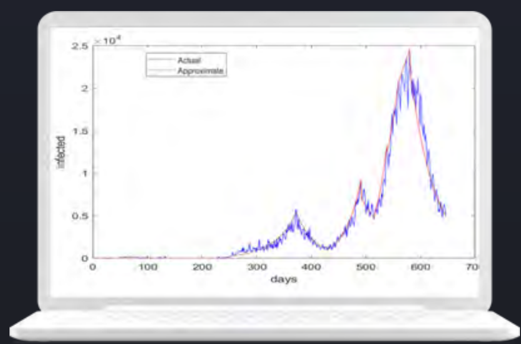
DIFFERENTIAL EQUATION

Susceptible, ds/dt = -βis
Infected, di/dt= βis- γi
Recovered, dr/dt= γi

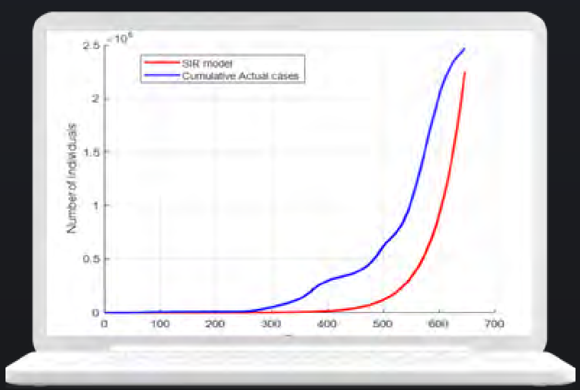
CUMULATIVE RECOVERED AND SIR MODEL



ACTUAL CASES AND APPROXIMATE DATA



CUMULATIVE INFECTED AND SIR MODEL



CONCLUSION

SIR models inconsistent because

- These oversimplified models neglect the factors that have a significant impact on illness progression.
- Assumptions that are not always true (system is always closed, recovered individuals are assumed as immunized)
- Changes of properties of virus Covid 19 (omicron,delta)

References

- Chicone, C. (2006). Ordinary Differential Equations with Applications (Texts in Applied Mathematics, 34) (2nd ed.) [E-book]. Springer.
- Ministry of Health. (2020, January 25). in Malaysia. COVIDNOW. https://covidnow.moh.gov.my/
- Burden, R. L., Faires, D. J., & Burden, A. M. (2015). Numerical Analysis (10th ed.) [E-book]. Cengage Learning.

Polynomial Domains are Unique Factorization Domains

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Abstract

A polynomial with coefficients in a particular field is expressed as the product of irreducible factors with coefficients in the same domain using polynomial factorization. In 1973, Theodor von Schubert published the first polynomial factorization algorithm and later rediscovered by Leopold Kronecker in 1882 where he extended Schubert's algorithm to the coefficient in an algebraic extension. In this project, we first introduce algebraic structures such as rings and fields and their properties. We then discuss polynomial rings including irreducible polynomial and primitive polynomial. We explore some types of integral domains namely, euclidean domains, principle ideal domains and also unique factorization domains. Lastly, we attempt to find different proof that polynomial domains are unique factorization domain.

1. Chapter ! : Rings and Fields

[1] In 1871, Richard Dedekind defined the concept of the ring of integers of a number field. However, Dedekind did not define the concept of a ring in general setting until Adolf Fraenkel presented it in 1915.

Binary operation : Let S be non-empty set. An operation $*$ on S is said to be a binary operation if a$*$b $\in$ S for any a.b $\in$ S.

Let R be a non-empty set, and (+), $(\cdot)$ be two operations on R.

(1) (R, +, $\cdot$) is an abelian group

(2) (a$\cdot$b)$\cdot$c = a$\cdot$(b$\cdot$c)

(3) a$\cdot$(b+c) = a$\cdot$b + a$\cdot$c

(4) (a + b)$\cdot$c = a$\cdot$c + b$\cdot$c

(5) a$\cdot$b = b$\cdot$a

(6) $\exists$1 $\in$ R such that a$\cdot$1 = a = 1$\cdot$a for $\forall$ a $\in$ R

(7) a$\cdot$b = 0 $\Rightarrow$ a=0 or b=0

(8) $\forall$ a $\in$ R/$\{0\}$, $\exists\, a^{-1} \in R$ such that $a \cdot a^{-1} = 1 = a^{-1}$$\cdot$a

(R, +, $\cdot$) is called

- A Ring if R satisfies (1) to (4)
- A Integral Domain in R satisfies (1) to (7)
- A Field if R satisfies (1) to (6) and (8)

Theorem 1.1. *$\mathbb{Z}_p$ is an integral domain if and only if p is a prime.*

Theorem 1.2 (Cancellation Law). *Let R be a commutative ring with identity. Then R is an integral domain if and only if $ab = ac \Rightarrow b = c$ for all $a \neq 0, b, c \in R$.*

Theorem 1.3. *Every field is an integral domain.*

Theorem 1.4. *Every finite integral domain is a field.*

2. Chapter 2 : Polynomial Rings

2.1 Polynomial Rings

Definition 2.1 (2). *Let R be a commutative ring. The polynomial ring in the indeterminate x with the coefficient from $\mathbb{R}$ is denoted R[x], that is*

$$R[x] = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 \quad (1)$$

$$= \sum_{i=0}^{n} a_ix^i, a_i \in \mathbb{R} \quad (2)$$

Two element, $\sum_{i=0}^{n} a_ix^n$ and $\sum_{i=0}^{m} b_ix^M$ are consider equal if and only if $a_i = b_i$ for all $i \in \mathbb{N}$.

Theorem 2.1. *If R is an integral domain, then R[x] is an integral domain.*

2.2 Irreducible polynomial

Definition 2.2. *Let D be an integral domain. A polynomial f(x) from D[x] that is neither the zero polynomial nor a unit in D[x] is said to be irreducible over D if, whenever f(x) = g(x)h(x), with g(x) and h(x) from D[x], then g(x) or h(x) is a unit in D[x].*

A nonzero, nonunit element of D[x] that is not irreducible over D is called reducible over D.

2.3 Primitive Polynomial

Let f $\in \mathbb{Z}[x]$ with

$$f(x) = a_0 + a_1x + \cdots + a_mx^m$$

then f is said to be a primitive polynomial over $\mathbb{Z}$ if gcd($a_0, a_1, \ldots, a_m$) = 1. Here, gcd($a_0, a_1, \ldots, a_m$) also known as the content of a nonzero polynomial.

Example 1. *Consider*

$$f(x) = 1 + 3x + 5x^2,$$
$$g(x) = 2 + 4x + 6x^2$$

Then, f is a primitive polynomial over $\mathbb{Z}$ since gcd(1,3,5) = 1 while g is not a primitive polynomial over $\mathbb{Z}$ since gcd(2,4,6) = 2.

Theorem 2.2 (Gauss's Lemma). *The product of two primitive polynomials is primitive.*

Theorem 2.3. *Let f(x) $\in \mathbb{Z}[x]$. if f(x) is reducible over $\mathbb{Q}$, then it is reducible over $\mathbb{Z}$.*

3. Chapter 3 : Domains

3.1 Euclidean Domains (EDs)

Let n be a function such that $n : R/\{0\} \to \mathbb{N} \cup \{0\}$. Here, n is called the norm function on R.

Definition 3.1. *An integral domain D is called a Euclidean domains with norm n such that for any a,b $\in$ D, there exist elements q and r in D such that a = bq + r, where r = 0 or n(r) < n(b). The element q is called the quotient and r is the remainder.*

Example 2. *The integer $\mathbb{Z}$ are Euclidean domains with the norm given by n(a) = $|a|$ for all a $\in \mathbb{Z}$.*

3.2 Principle Ideal Domains (UFDs)

Let R be a commutative ring. An ideal of the form aR = ar : r $\in$ R is called a Principle Ideal generated by a which is denoted as

$$< a > := \{ar : a \in I, r \in R\}.$$

Definition 3.2. *Let R be an integral domain. Then R is said to be a Principle Ideal Domains (PIDs) if every ideal R is a principle ideal.*

Example 3. *$\mathbb{Z}$ is a PIDs.*

Example 4. *$\mathbb{Z}[x]$ is not a PIDs*
The ideal (2,x) is not principle. IF (2,x) is principle, say $(2, x) = ((f(x_1))$, then deg f(x) > 0: a contradiction since 2 is not multiple of f(x).

3.3 Unique factoriation Domains (PIDs)

Definition 3.3. *Let R be an integral domain. Then R is UFDs if the following properties hols,*

1. *Every element a $\in$ R that is nonzero and that is not a unit can be expressed as a product of irreducible elements in R.*
2. *If $a = p_1p_2 \ldots p_m$ and $a = q_1q_2 \cdots q_n$ where $p_1, p_2, \cdots, p_m, q_1, q_2, \cdots, q_n$ are irreducible elements in R then m = n and the factors of both expressions can be arranged so that $p_i \sim q_i$ for each $i \in \{1, 2, \cdots, m\}$.*

4. Chapter 4: Polynomial Domains are Unique Factorization Domains

4.1 First Attempt

Definition 4.1. *A ring R is noetherian if every ideal of R is finitely generated*

Definition 4.2. *Let R be a ring. An ascending chain of ideals in R is a collection of ideals that is,*

$$I_1 \subseteq I_2 \subseteq \ldots.$$

We say that the above chain stabilize if there exist integer n such that $I_n = I_{n+1} = I_{n+2} = \ldots$. In other word, $I_m + I_{m+1}$ for all $m \geq n$.

Proposition 4.1. *A ring R is notherian if and only if every ascending chain of ideals in R are stabilizes.*

Proposition 4.2. *A PIDs is noetherian.*

Lemma 4.1. *Let R be a PIDs and let $I_1, I_2, \ldots$ be ideals of R such that $I_1 \subseteq I_2 \subseteq \cdots \subseteq I_n \subseteq \ldots$, there exist a positive integer m such that $I_n = I_m$ for all $m \geq n$.*

Theorem 4.1. *Every PIDs is UFDs.*

Proof. For the existence of the theorem, let a be a nonzero and nonunit element in PIDs. If a is irreducible, we are done. If not, we write $a = a_1a_2$ for a unit a_1, a_2. If a_2 is not irreducible, then $a_2 = a_3a_4$ which implies $a = a_1a_3a_4$. If a_4 is not irreducible, then $a_4 = a_5a_6$ and so $a = a_1a_3a_5a_6$. Eventually $a = a'p$ where p is irreducible. If the process does not terminates, we have $< a > \subsetneq < a_2 > \subsetneq < a_4 > \subsetneq \ldots$; a contradiction from the Lemma 4.1.1. Therefore, $a = a'p_1 = a''p_2p_1 = p_mp_{m-1} \cdots p_2p_1$.

To prove the uniqueness of the theorem, suppose $a = p_1 \cdots p_n = q_1 \cdots q_m$ where all are irreducible. Then, p_1 divides q_i for some $i \in \{1, 2, \ldots, m\}$. Thus $p_1u = q_i$ for some unit u and so p_1 and q_i are associate. Since element a is an integral domain, then cancellation holds here. Then, $p_2 \cdots p_n = uq_1 \cdots q_{i-1}q_{i+1} \cdots q_m$. Continue inductively, then we have that the factorization is unique. □

Theorem 4.2. *Let $\mathbb{F}$ be a field, then $\mathbb{F}[x]$ is a PIDs.*

Proof. We know that $\mathbb{F}[x]$ is an integral domain. Let I be an ideal. If $I = 0$, then $I =< 0 >$. Suppose $I \neq 0$ and let $g \in I$ be nonzero polynomial of minimal degree . Claim that $f =< g >$. Suppose $f \in I$. By division algorithm, there are nonzero polynomial q and r such that $f = qg + r$ and either r = 0 or deg r < deg g. Since $f, g \in I$, r = f - qg $\in$ I. Since g is minimal degree in I, we must have r = 0. Thus, $f = q.g \in < g >$. If $I \neq 0$ and $f \in I$ is of minimal degree, then f is a minimal polynomial of I an $I =< f >$. Therefore, $\mathbb{F}[x]$ is PIDs. □

4.2 Second Attempt

Proposition 4.3. *Let $f(x) \in \mathbb{Q}[x]$, deg f > 0. Then f can be written uniquely as a product $f = cf_0$ where $c \in \mathbb{Q}$ and $f_0 \in \mathbb{Z}[x]$ is primitive.*

Proposition 4.4. *Let $f, g \in \mathbb{Z}[x]$ with f primitive. If f divides g in $\mathbb{Q}[x]$, then f divides g in $\mathbb{Z}[x]$.*

Proposition 4.5. *Let $f(x) \in Z[x]$ be an irreducible polynomial with positive leading coefficient. Then one of the following holds :*

1. *if deg f = 0, then $f(x) \in \mathbb{Z}$ is a prime integer.*
2. *if $deg f > 0$, then f(x) is a primitive polynomial and f(x) is irreducible in $\mathbb{Q}[x]$.*

Theorem 4.3. *$\mathbb{Z}[x]$ is a UFDs.*

Proof. Let f$\in \mathbb{Z}[x]$ be irreducible

Case 1: Suppose $f \in \mathbb{Z}$ and f is a prime integer called p. Consider f divides gh for some $g, h \in \mathbb{Z}[x]$ such that $g = cg_0$ and $h = dh_0$ where c is the content of g, d is content of h and g_0, h_0 are the primitive polynomial. Then we have f divides $cdg_0h_0 = gh$. Since g_0, h_0 are primitive polynomial, by Gauss Lemma, g_0h_0 is also primitive. Then, there is a coefficient, say y of g_0h_0 such that p does not divide y. But, p divides cdg_0h_0 implies p divides ycd. It follows that p divides cd which then either p divides c or p divides d. Next, we have p divides cg_0 or p divides dh_0. Therefore, p divides g or p divides h. Hence f is prime in $\mathbb{Z}[x]$. Thus, $\mathbb{Z}[x]$ is a UFDs.

Case 2: Suppose $deg > 0$, then f(x) is primitive and irreducible. Let f divides gh in $\mathbb{Z}[x]$ where $g, h \in \mathbb{Z}[x]$. Then, f divides gh in $\mathbb{Q}[x]$. Since $f \in \mathbb{Q}[x]$ is irreducible in $\mathbb{Q}$ and $\mathbb{Q}[x]$ is a PIDs, hence $\mathbb{Q}[x]$ is a UFDs and so f is prime in $\mathbb{Q}[x]$. It follows that, either f divides g in $\mathbb{Q}[x]$ or f divides h in $\mathbb{Q}[x]$. Then, we have f divides g in $\mathbb{Z}[x]$ or f divides h in $\mathbb{Z}[x]$. Hence f is prime in $\mathbb{Z}[x]$. Thus, $\mathbb{Z}[x]$ is a UFDs. □

References

[1] Hosch, W. L. (2009, August 18). *ideal. Encyclopedia Britannica.*

[2] Polynomial Rings. (n.d.). Retrieved December 8, 2021 from https://sites.millersville.edu/bikenaga/abstract-algebra-1/polynomial-rings/polynomial-rings.html

PERFORMANCE ANALYSIS OF SUCCESSIVE OVER RELAXATION (SOR) METHOD IN DETERMINING PEAK JUNCTION TEMPERATURE OF SEMICONDUCTOR DEVICE

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ABSTRACT

In order to improve device reliability and make operating life last longer, many microelectronic devices have been derating junction temperature. If the junction temperature exceeds its limit, it can damage the electronic devices in an undesirable manner. Therefore, as a solution we can use thermal control system to achieve high performance of electronic system. The numerical approach to predict the peak junction temperature of semiconductor devices will be used to discretize the heat conduction equation. In the first step, Crank-Nicolson method finite difference will be used to discretize the heat conduction equation. Next, Successive Over Relaxation (SOR) and Gauss-Seidel (GS) iterative methods will be applied to solve the generated system of linear equations. Based on results obtained, it clearly shows that SOR method gives less number of iteration and computational time compared to the conventional GS iterative method. However, in terms of accuracy of peak junction temperature, both tested methods are comparable.

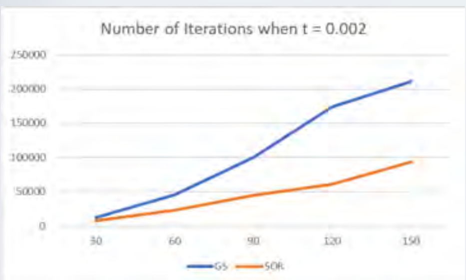
OBJECTIVE

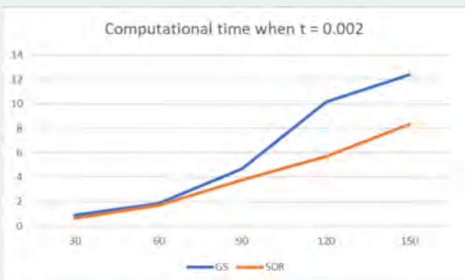
1)To discretize the heat conduction equation by using Crank Nicolson numerical scheme.
2)To apply SOR method in solving the generated system of linear equation from heat conduction equation.
3)To compare the performances of SOR method with GS method to solve the peak junction temperature.

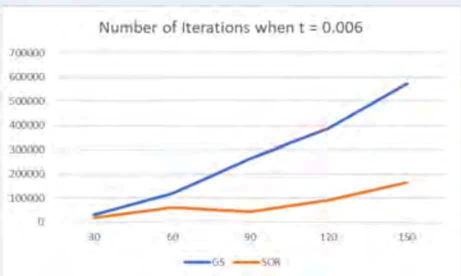
INTRODUCTION

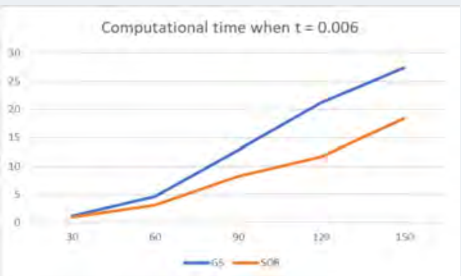
Junction temperature is the highest operating temperature of semiconductor device. In order to determine the reliability and performance of semiconductor devices and power devices reliability, junction temperature is a very important parameter to be measured. In this study, to monitor the temperature of semiconductor, mathematical modeling is used to predict IC junction temperature. One-dimensional heat conduction equation will be used to predict IC junction temperature.

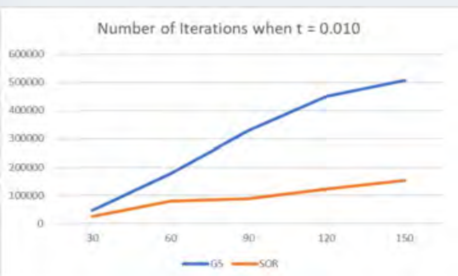
RESULT

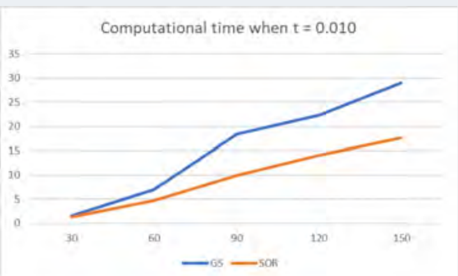












METHODOLOGY

First, the heat conduction equation will be discretize using Crank Nicolson method. Then, it will form a system of linear equation. After that, iterative methods will be applied to find the peak junction temperature. The Iterative method that use is SOR and GS method. Both methods will be compared to find the best effectiveness.

CONCLUSION

In conclusion, this research is focus on finding peak junction temperature of semiconductor devices on Printed Circuit Board (PCB). The heat conduction equation is used in solving the thermal systems. Therefore, the results of this studies as follows:
1)The Crank-Nicolson method successfully applied in the heat conduction equation to form a linear system after discretize using that method. A linear system is solved using GS and SOR method.
2)The SOR method is successfully applied in solving the generated system of linear equation heat conduction equation.
3)The SOR method can give less number of iterations and computational time compared to GS method and both methods can give same maximum temperature.
Therefore, in determining the peak junction temperature of semiconductor devices, the SOR method is more efficient compared to GS method.

REFERENCES

1) Ammous, A., Ghedira, S., Allard, B., & Morel, H. (n.d.). Choosing a thermal model for electrothermal simulation of Power Semiconductor Devices. PESC 98 Record. 29th Annual IEEE Power Electronics Specialists Conference (Cat. No.98CH36196). https://doi.org/10.1109/pesc.1998.703403
2) Andresen, M., Ma, K., Buticchi, G., Falck, J., Blaabjerg, F., & Liserre, M. (2018). Junction Temperature Control for more reliable power electronics. IEEE Transactions on Power Electronics, 33(1), 765–776. https://doi.org/10.1109/tpel.2017.2665697
3) Babajide Johnson, O., & Idowu Oluwaseun, L. (2020). Crank-Nicolson and modified Crank-Nicolson scheme for one dimensional parabolic equation. International Journal of Applied Mathematics and Theoretical Physics, 6(3), 35. https://doi.org/10.11648/j.ijamtp.20200603.11
4) Babajide Johnson, O., & Idowu Oluwaseun, L. (2020). Crank-Nicolson and modified Crank-Nicolson scheme for one dimensional parabolic equation. International Journal of Applied Mathematics and Theoretical Physics, 6(3), 35. https://doi.org/10.11648/j.ijamtp.20200603.11
5) Dessalew, G., Kebede, T., Awgichew, G., & Walelign, A. (2021). Generalized refinement of Gauss-Seidel method for consistently ordered 2-cyclic matrices. Abstract and Applied Analysis, 2021, 1–7. https://doi.org/10.1155/2021/8343207

MODELLING TRANSMISSION OF COVID-19 USING AN AGENT-BASED MODEL

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ABSTRACT

We study the spread of COVID-19 and simulate the transmission of COVID-19 within the campus depending on various behaviors and movements of people and numerous variables that may affect the outcomes of the simulation that can provide a theoretical contingency plan for the authority. The simulation model was based on a simple susceptible-infected-recovered (SIR) model with a total population that is not kept constant depending on the scenario of the simulation. By comparing the data and outcomes from these different scenarios, we can conclude that by implementing social distancing and various countermeasures with full obligation from people and strong policies from the authority, the spread of COVID-9 can be curbed and be under control.

INTRODUCTION

COVID-19 is very infectious disease, and it can spread to a scale that very unpredictable and deadly to human's survival. The uncertainty of the spread of COVID-19, its evolution and how it affects the daily basis of the people makes it really hard for the authority to make it a priority at the beginning, especially with the difficult trade-offs given the health, economic and social challenges it may raises and eventually worsen.

COVID-19, according to the report, was an "awful wake-up call" and "the Chernobyl moment of the 21st century." It claims that the current system is unsuitable to prevent the spread of a pandemic caused by a novel and highly contagious bacterium that might arise at any time. From here, we able to deduce that much faster response from the authority is needed as the COVID-19 virus has been discovered. Thus, we need to able to predict the spread of COVID-19 with current conditions and trajectory. For that, our study will use agent-based model to showcase the spread of COVID-19 and obtain the outcomes of the spread so future planning can be done and a full contingency plan can be prepared before the outbreak happens.

OBJECTIVES

a) To investigate the spread of COVID-19 on campus by incorporating the random movement of people.
b) To study the effectiveness of control measures in curbing the spread of COVID-19.
c) To provide a contingency plan to authority in controlling the spread of COVID-19.

METHODOLOGY

We use GAMA platform as our simulation platform. The model is based on three main agent profiles which is Susceptible, Infected and Recovered. Agent will have their profile changed in every simulation step based on the probability for them to get infected and recovered.

Initial parameters:

- infectivity rate = 1.00
- recovery rate = 0.90
- infection range = 2 meters
- duration of the simulation = 200 days
- population = 20000 people with 10 infected agents
- area of study = Universiti Malaya
- speed movement of agents = 2-30km per hour

ASSUMPTIONS

- Tendency for agent to leave from initial position = 100%
- Their movement are based on travelling on roads only.
- Infection happens when the agents are within infection range with the infected agents
- Recovered agents can be infected again (multiple infections).
- For multiple infection, the probability is set between 0.17 and 0.23.
- The recovery rate will stay constant at 90%.
- The control measures used in the simulations are not specific.
- The infectivity rate is halved after implementing control measures.

RESULTS

The number of infected and recovered was recorded from every single simulation step and a display of agents, buildings and roads with a graph will be produced as visual representation.

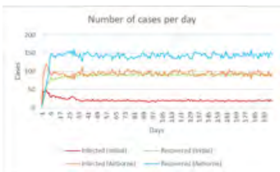
Susceptible   Infected

GRAPHS

The graphs showed the outcomes of different scenarios with different parameter values (infection by airborne, implementing social distancing and control measures) and comparing them with initial condition (infection by close contact, no social distancing and control measures).

As we can see, infection by airborne are much deadly and greater compared to infection by close contact. Implementing social distancing and control measures are able to reduce the total number of cases until it reaches zero.

Airborne



Much wider infection range means more people or agents able to interact to each other. This will allow the infection to become more pronounced and will spread much faster to more people by day. Because of that, the infected cases will stay high even with high recovery rate.

Social distancing

Less population means much lower population density. People are not confined, and common areas can be much less crowded. This will reduce the number of people that may be within close proximity with the infected ones hence reduce the probability for people to get infected.

Control measures

Control measures are being proven as one of the best ways to prevent the infectivity of COVID-19 from staying high. Therefore, with much lower infectivity rate and high recovery rate, people can recover much faster compared to get infected.

CONCLUSION

COVID-19 can be spread to people, either close proximity or by airborne. Thus, by implementing several control measures including social distancing as soon as possible can lessen the impact of infection thus reduce the daily COVID-19 cases until non-existent. However, this result only can be achieved if the control measures are being implemented with full authority and complete obedience.

Therefore, more extreme moves should be taken by the authority to ensure that the community do follow the rules in order to curb the spread of COVID-19 to the fullest.

References

1. Allain-Dupré, D., Chatry, I., Kornprobst, A., &; Michalun, M.-V. (n.d.). The territorial impact of covid-19: Managing the crisis across levels of government. OECD. Retrieved February 3, 2022, from https://www.oecd.org/coronavirus/policy-responses/the-territorial-impact-of-covid-19-managing-the-crisis-across-levels-of-government-d3e314e1/
2. Niazi, M., &; Hussain, A. (n.d.). Agent-based Computing from Multi-agent Systems to Agent-Based Models: A Visual Survey. Agent-based model explained. Retrieved February 3, 2022, from https://everything.explained.today/agent-based_models/
3. Güner, R., Hasanoğlu, I., & Aktaş, F. (2020, April 21). Covid-19: Prevention and control measures in community. Turkish journal of medical sciences. Retrieved February 7, 2022, from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7195988/

Field and Algebraic Extension

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Abstract

A field extension over F is a field E where $E \supseteq F$. In this dissertation, we discuss algebraic structures such as groups, rings, and fields. We introduce algebraic and transcendental field extensions and their properties. We see how algebraic field extensions are constructed from irreducible polynomials. We will also look at field embeddings and automorphisms between extension fields.

Simple Extension

A vector space V over F is an Abelian group with a good concept of scalar multiplication by F. If we have an extension fields of K/F, then we may naturally regard K as a vector space over F. This is because there is a natural concept of scalar multiplication on K by F. The properties of a vector space are automatically satisfied by the ring axiom for K. We are then able to use such notions as dimension and basis in our discussion.

Definition: An element u of an extension field K of F is said to be algebraic over F if u is the root of some nonzero polynomial in $F[x]$. If it is not the root of any nonzero polynomial in $F[x]$, then it is transcendental over F.

Example: Let $u = \sqrt{2} + 1 \in \mathbb{R}$. It is algebraic over $\mathbb{Q}$,with degree 2 since it satisfy $(u-1)^2 - 2 = 0$

Example: The real number π and e are transcendental over $\mathbb{Q}$.

Algebraic Extension

Definition: An extension field K of field F is said to be algebraic extension of F if every element of K is algebraic over F.

Example: If $a + bi \in \mathbb{C}$, then $a + bi$ is a root of $(x-(a+bi))(x-(a-bi)) = x^2 - 2ax + (a^2+b^2) \in R[x]$

Therefore, $a + bi$ is algebraic over R and hence, $\mathbb{C}$ is an algebraic extension of R. However, neither $\mathbb{C}$ nor $\mathbb{R}$ is an algebraic extension of $\mathbb{Q}$ since there are real number (such as π and e) that are not algebraic over $\mathbb{Q}$

Field Embedding

Definition: Let K be a number field. Then, an embedding of K is a non-zero field homomorphism $\sigma : K \to \mathbb{C}$.
Remark: Note that since σ is a field homomorphism we must have $\sigma(x) = x, \forall x \in \mathbb{Q} \subset K$. Note that since our number field are always assumed to be contained on $\mathbb{C}$, we always have at least one embedding, called the identity embedding.(i.e. $K \subset \mathbb{C}$ such that $\sigma : K \to \mathbb{C}$, $\sigma(\alpha) = \alpha \ \forall \alpha \in K$)

Proposition: Let K/F be an extension of number field and $\sigma : K \to \mathbb{C}$ be an embedding such that $\sigma|_F = e$, where e is the identity. Then if $f(x) \in F[x]$ is an irreducible polynomial and α is one of its roots, then σ sends α to a conjugate of α.
Let $f(x) = a_0 + a_1x + \ldots + a_nx^n$. Then, since σ fixes F and since $a_0 + a_1\alpha + \ldots + a_n\alpha^n = 0$ for $a_i \in F$, it follows that:

$$\begin{aligned}\sigma(a_0 + \ldots + a_n\alpha^n &= \sigma(a_0) + \sigma(a_1)\sigma(a_1)\sigma(\alpha) + \ldots + \sigma(a_n)\sigma(\alpha)^n \\ &= a_0 + a_1\sigma(\alpha) + \ldots + a_n\sigma(\alpha)^n = 0\end{aligned}$$

Therefore, $\sigma(\alpha)$ and α both satisfy the same polynomial so, they are conjugate.

Important Theorems in Algebraic Extension

Theorem 1: If K is a finite extension of F, then K is an algebraic extension of F.

Proof: Suppose that $[K : F] = n$ and $a \in K$. Then the set$\{1, a, \ldots, a^n\}$ is linearly independent over F, where there are elements$c_0, c_1, \ldots, c_n$ in F, not all zero such that $c_na^n + c_{n-1}a^{n-1} + \ldots + c_1a + c_0 = 0$ Clearly, a is a zero of the nonzero polynomial $f(x) = c_nx^n + c_{n-1}x^{n-1} + \ldots + c_1x + c_0$

Theorem 2: Let $K = F\{u_1, ..., u_n\}$ is a finitely generated extension field of F, and each u_i is algebraic over F, then K is a finite dimensional algebraic extension of F

Proof: The field K can be obtained from this chain of extensions: $F \subseteq F(u_1) \subseteq F(u_1, u_2) \subseteq F(u_1, u_2, u_3) \subseteq \ldots\ldots \subseteq F(u_1, \ldots, u_{n-1}) \subseteq F(u_1, \ldots, u_n) = K$ Furthermore, $F(u_1, u_2) = F(u_1)(u_2), F(u_1, u_2, u_3) = F(u_1, u_2)(u_3)$ and in general $F(u_1, \ldots, u_i$ is the simple extension $F(u_1, \ldots, u_{i-1})(u_i)$. Each u_i is algebraic over$F$ and hence, algebraic over $F(u_1, \ldots, u_{i-1})$. But every simple extension by an algebraic element is finite dimensional. Therefore, $[F(u_1, \ldots, u_i) : F(u_1, \ldots, u_{i-1})]$ is finite for each $i = 2, \ldots, n$. Consequently, by repeated application of the previous theorem, we see that $[K : F]$ is the product $[K : F(u_1, \ldots, u_{n-1}] \ldots [F(u_1, u_2 : F(u_1)][F(u_1) : F]$. Thus $[K : F]$ is finite and hence K is algebraic over F.

Field Automorphism and Galois group

Definition: Let F be a field. A field automorphism of F is a bijecion $\phi : F \to F$ such that for all $a, b \in F$ $\phi(a + b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$
To put it another way, the ϕ field's structure must be preserved.

Proposition: Let ϕ be an automorphism of an extension field F of $\mathbb{Q}$, then $\phi(q) = q$ for all $q \in \mathbb{Q}$

Proof: Suppose that $\phi(1) = q$. Clearly, $q \neq 0$. This is because $q = \phi(1) = \phi(1 \cdot 1) = \phi(1)\phi(1) = q^2$ Similarly, $q = \phi(1) = \phi(1 \cdot 1 \cdot 1) = \phi(1)\phi(1)\phi(1) = q^3$ and so on. It follows that $q^n = q$ for every $n \geq 1$. Thus, $q = 1$

Definition: Let F be an extension field of $\mathbb{Q}$. The Galois group of F is the group of automorphisms of F, denoted Gal(F).

References

[1] Hungerford, T. W. (2012). *Abstract algebra: an introduction*. Cengage Learning.

[2] Gallian, J. A. (2021). *Contemporary abstract algebra*. Chapman and Hall/CRC.

INTRODUCTION TO WORLDWIDE EARTHQUAKE PROBABILITY DISTRIBUTIONS

INTRODUCTION

Over the year, Poisson distribution has been the most common distribution used in modelling earthquake data because of its simplicity and easy to use. However, because of the variety in earthquake data and the temporal relationships that are common in many real earthquake sequences, the Poisson distribution seems to be ineffective. The statistical goodness-of-fit tests on worldwide seismicity data from 1921 to 2021 are presented in this work. The tests reveal that earthquake temporal occurrences do not always match the Poisson distribution, which is often used in earthquake research. The Negative Binomial distribution, on the other hand, was found to be a good model for capturing observed earthquake magnitude distributions with statistical significance.

OBJECTIVE

The study aims to achieve the following objectives:

1) to investigate that earthquake temporal occurrences do not necessarily follow the Poisson distribution even though it has been commonly applied to earthquake studies.

2) to make a comparison between Poisson and Negative Binomial distribution in which better to represent earthquake data.

METHODOLOGY

POISSON DISTRIBUTION

According to the textbook [2], the probability distribution function of the Poisson distribution is as follows:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0,1,2 \ldots,$$

Where λ is a poisson parameter which is the mean annual rate.

NEGATIVE BINOMIAL DISTRIBUTION

According to G. Bazigos (1981), by using recurrence formula, the individual terms of the series are,

$$P(0) = q^{-k} = (1 + \frac{m}{k})^{-k}$$

While

$$P(x+1) = (\frac{k+x}{x+1})(\frac{m}{m+k})P(x)$$

The parameters of the distribution are the arithmetic mean $(m)$and the exponent k.

CHI-SQUARE TEST

We use Chi-Square test to estimate how closely our observed distribution matches an expected distribution. According to the textbook [2], chi-square value is calculated as follows:

$$\chi^2{}_{(k-1)} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

Where χ^2 is the chi-square value, k is the degree of freedom, O_i is the observed frequency and E_i is the fitted/expected frequency.

RESULT AND DISCUSION

Mag	Min	1st Qu.	Median	Mean	3rd Qu.	Max	StandardDev.	Variance
6.5	18	32	39	39.51	46	66	10.58	111.91
6.6	15	24	31	31.16	38	53	8.37	70.03
6.7	12	19	25	25.18	30	42	6.77	45.87
6.8	7	16	19	20.19	24	19	5.65	31.95

From the Table we observed that the variance is larger than the mean for all magnitude. Hence overdispersion occur.

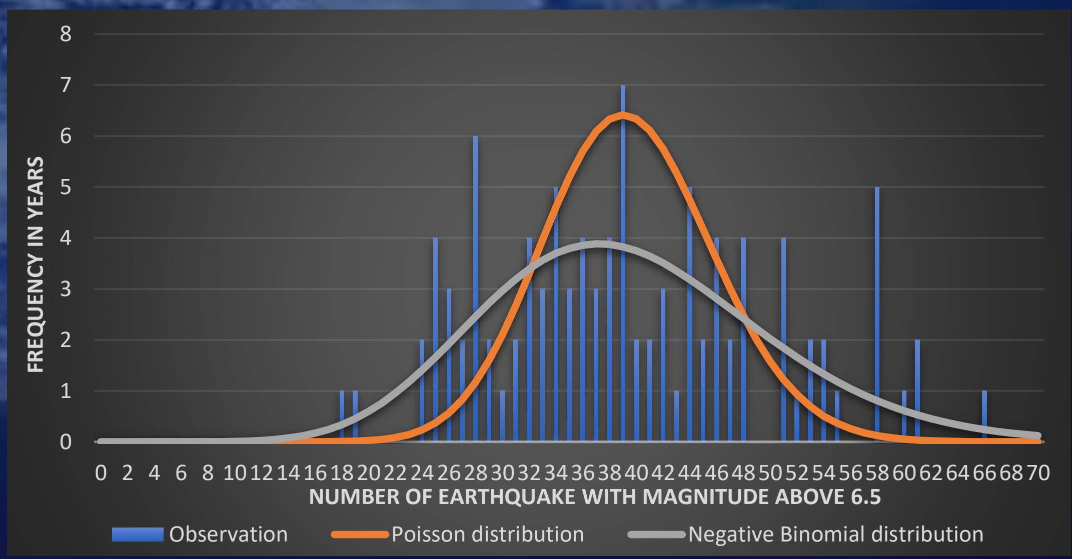

Figure shows the comparison between two different model and observed frequency for magnitude above 6.5. Clearly, we can see that NB distribution have a greater spread than a Poisson distribution and the graph have lower disparity compared to Poisson.

Mag	Annual rate	DF	Poisson distribution			Negative Binomial distribution			
			Chi square	critical value	Model accepted/ rejected	K-value	Chi square	critical value	Model accepted/ rejected
6.5	39.5	70	1003.1	90.5	Rejected	21.6	68.2	90.5	Accepted
6.6	31.2	60	239.7	79.1	Rejected	25.0	40.5	79.1	Accepted
6.7	25.2	50	121.7	67.5	Rejected	30.6	36.0	67.5	Accepted
6.8	20.2	40	99.6	55.8	Rejected	34.6	32.0	55.8	Accepted

From table above, the negative binomial distribution can well describe the earthquake occurrences, with sufficient statistical significance for the hypothesis – earthquake occurrences distribution follows the negative binomial distribution – **not being rejected** by chi-square tests. By contrast, using Poisson distribution few magnitudes seem to reject this distribution by chi-square test.

CONCLUSION

The observed earthquake occurrence distribution can be well described by the negative binomial distribution. As a result, the negative binomial distribution is a more statistically significant alternative to compare the Poisson distribution for modelling earthquake occurrence distributions. Thus, it is highly suggested to used Negative Binomial distribution in modelling earthquake occurrence for future studies.

REFERENCES

[1] (1 1, 2022). Retrieved From Earthquake Catalog United State Geological Survey (USGS): Https://Earthquake.Usgs.Gov/Earthquakes/Search/

[2] Hogg, R. V., Tanis, E. A., & Zimmerman, D. L. (2015). Probability And Statistical Inference. United States Of America: Pearson Education.

[3] Cornell, C.A., 1968, Engineering Seismic Risk Analysis: Bulletin Of The Seismological Society Of America, V. 58, No. 5, P. 1583–1606.

SIN3015 MATHEMATICAL SCIENCE PROJECT

UNIVERSITI MALAYA

MODELLING THE EFFECT OF COVID VACCINES WANING IN THE SPREAD OF DISEASE

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ABSTRACT

A new compartmental model is proposed for studying the novel coronavirus (COVID-19) spread in Malaysia by incorporating the effect of the COVID vaccine waning. Considering that there is a possibility for a person who has got vaccinated to get infected and thus spread the virus depending on the vaccine effectiveness. Furthermore, a mathematical analysis is carried out to find the epidemic equilibrium and the basic reproduction number, Ro of the proposed model. The impact of various embedded parameters such as the transmission rate, vaccination program, outcome of the percentage of the vaccinated population and the type of the vaccines wanes over time were explained in the numerical results. Then, the Nelder-Mead algorithm is used as a method in optimization with real data is conducted and the prediction on transmission trajectory of COVID-19 with the effect of vaccine waning was produced. By comparing all the data obtained, we can conclude that although getting vaccinated against COVID-19 can continue to reduce the risk of infections and death, a COVID booster is recommended for COVID vaccines that have a fast-waning rate. Through this paper, herd immunity is expected to achieve when 80% to 90% of people are needed to be vaccinated.

OBJECTIVES

a) To propose a mathematical model forecasting transmission of COVID-19 in Malaysia by incorporating the effect of COVID vaccine waning.
b) To study the impact of vaccination programs on COVID-19 outbreaks.
c) To investigate the percentage of people who need to be immune against covid-19 disease in order to achieve the herd immunity.

INITIAL MODEL PARAMETERS AND INTERPRETATIONS

Parameter	Initial value	Description	Reference
N	32657400	Total population size	[1]
α	0.00278	Vaccination rate	[1]
β	0.25	Transmission rate	Assumed
σ	0.11	Vaccine inefficacy (Pfizer)	[3]
λ^{-1}	7 days	Average latent time	[5]
κ_s	0.013	Death rate of susceptible	[1]
κ_v	0.007	Death rate of vaccinated	[1]
μ^{-1}	10 days	Average days until recovery	[6]
τ^{-1}	28 days	Average days until death	[7]

Table 1. Initial model parameters and interpretations.

MODEL FRAMEWORK

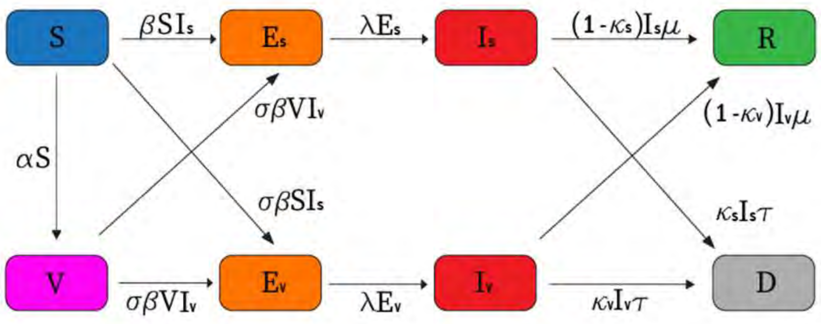

Figure 1. The flow chart of the disease transmission

MODEL EQUATIONS

The disease transmission in Figure 1 can be expressed in the form of nonlinear ordinary differential equations as below

$$\begin{aligned}
\frac{dS}{dt} &= -\frac{\beta S I_s}{N} - \alpha S - \frac{\sigma\beta S I_s}{N} \\
\frac{dE_s}{dt} &= \frac{\beta S I_s}{N} + \frac{\sigma\beta V I_v}{N} - \lambda E_s \\
\frac{dI_s}{dt} &= \lambda E_s - (1-\kappa_s) I_s \mu - \kappa_s I_s \tau \\
\frac{dR}{dt} &= (1-\kappa_s) I_s \mu + (1-\kappa_v) I_v \mu \qquad (1) \\
\frac{dD}{dt} &= \kappa_s I_s \tau + \kappa_v I_v \tau \\
\frac{dV}{dt} &= \alpha S - \frac{2\sigma\beta V I_v}{N} \\
\frac{dE_v}{dt} &= \frac{\sigma\beta S I_s}{N} + \frac{\sigma\beta V I_v}{N} - \lambda E_v \\
\frac{dI_v}{dt} &= \lambda E_v - (1-\kappa_v) I_v \mu - \kappa_v I_v \tau
\end{aligned}$$

MODEL ASSUMPTIONS

1) The total size of population remains constant where N = S +Es +Is +R +D +V +Ev +Iv.
2) The population are mix homogeneously.
3) A constant population was assumed due to the the short time period for the model development and projection, wherein changes of birth and death rates would be negligible.
4) The death rate is constant in time.
5) Th probability of being infected does not depend on factors such as age, gender or social status.
6) The asymptomatic and symptomatic infection is ignored.
7) The risk of virus mutation or the emergence of variants that partly evade vaccine is negligible.

THE EPIDEMIC EQUILIBRIUM AND THE BASIC REroDUCTION NUMBER, R0

The epidemic equilibrium X^0 of system (1) is obtained by setting all the derivatives to zero with $I = 0$,

$X^0 = (N-K, 0, 0, 0, 0, K, 0, 0)$, where $S = N - K$ and $V = K$

R_0 is computed by using the next-generation of matrix concept [2].

Therefore, the basic reproduction formula considering a large domain matrix is

$$R_0 = \frac{\beta S}{N[(1-\kappa_s)\mu + \kappa_s \tau]} + \frac{\sigma\beta V}{N[(1-\kappa_v)\mu + \kappa_v \tau]} \text{ , where } S = N - K \text{ and } V = K$$

RESULTS AND DISCUSSION

Since our model is a nonlinear equation, it might be impossible to use an analytical method to solve it. However, in the numerical simulations section, we decided to use ode15s in MATLAB as a numerical method for this project.

The impact of vaccination program

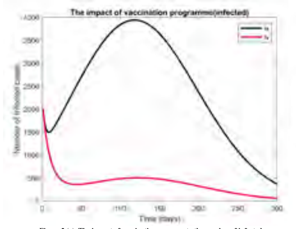

Figure 2 (a). The impact of vaccination program to the number of infected cases

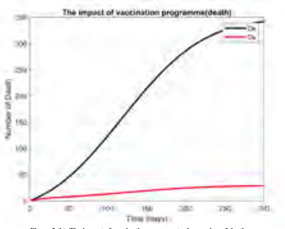

Figure 2 (b). The impact of vaccination program to the number of deaths

In Figure 2 (a), we notice that the impact of the vaccination program does show a significant decrease in the number of infected cases from vaccinated. On the day of the 100th, we observe a reduction of 87.39% for those who have been vaccinated compared to the susceptible people. The same goes for the number of cumulative deaths in Figure 2 (b), where there is a reduction of 91.55% on the day of the 300th for those who have been vaccinated. Therefore, from the results obtained, we can conclude that COVID-19 are effective at preventing infection, serious illness, and death. Vaccination can help protect those who are vaccinated as well as those around them by reducing the transmission of disease.

The impact of transmission rate

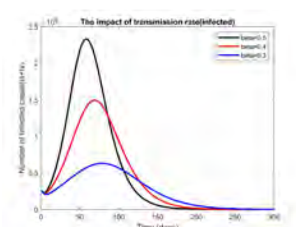

Figure 3 (a). The impact of transmission rate to the number of infected cases

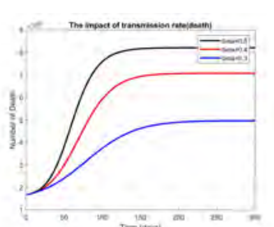

Figure 3 (b). The impact of transmission rate to the number of deaths

The quantitative analysis in Figure 3 (a) can be explained by comparing the highest number of infected cases for = 0.5 with = 0.4 and 0.3. We observed there is a reduction of 35.87% in the number of infected cases for = 0.4 while = 0.3 shows a reduction up to 72.91%. Meanwhile, on the last day of the simulations (300th days) in Figure 3 (b), there is a reduction of 13.81% and 39.54% in the number of cumulative deaths for = 0.4 and 0.5 respectively. In addition, the number of cumulative deaths in Figure 3 (b) shows a constant trend or have the same output for a certain time frame due to the decreased number of infected cases in figure 3 (a). Therefore, we can conclude that the lower the rate of transmission, the lower the risk to get infected by COVID-19 and thus reducing the number of cumulative deaths. Practising preventive measures such as physical or social distancing, quarantining, covering coughs and sneezes, hand washing, and keeping unwashed hands away from the face can minimize the risk of transmissions.

The outcome of the vaccinated population achieved for COVID-19 disease

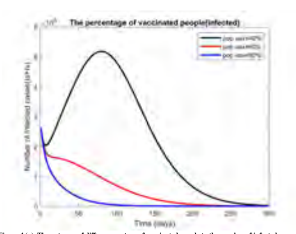

Figure 4 (a). The outcome of differ percentage of vaccinated people to the number of infected cases

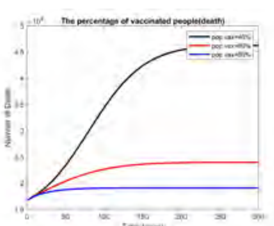

Figure 4 (b). The outcome of differ percentage of vaccinated people to the number of deaths

From the simulation results obtained in Figure 4 (a), on the day of the 100th, we observed that when the vaccinated people have reached 60% of the population, there is a reduction of 85.61% in the number of infected cases and when the number of vaccinated have reached 80% of the population, there is a 99.14% reduction from the 40% vaccinated population. The same goes in Figure 4 (b), where we noticed a decline in the number of deaths due to an increase in the percentage of vaccinated people. On the last day of the simulations (300th day), the 60% vaccinated people have reduced to 48.25% number of deaths which is lower compared to 58.83% for the 80% vaccinated people. Therefore, we can conclude that the total percentage of vaccinated people are needed to achieve herd immunity is estimated at around 80% to 90%. However, this only can be achieved if the viruses are not mutated or the emergence of variants that partly evade vaccine-induced antibodies.

The effect of vaccine wanes over time

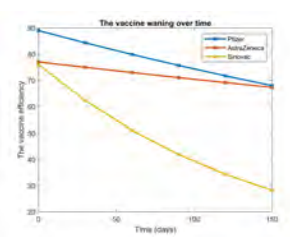

Figure 5 (a). The three types of vaccines wanes over time [3,4]

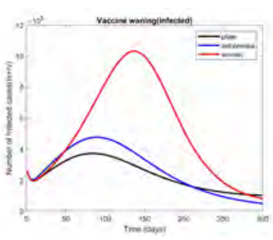

Figure 5 (b). The effect of COVID vaccines wanes to the infected cases

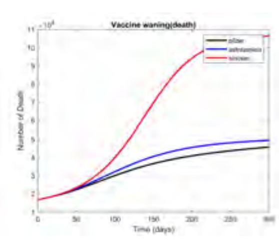

Figure 5 (c). The effect of COVID vaccines wanes to the number of deaths

In this section, we would like to focus on three types of vaccines that are mostly used in Malaysia for vaccination drive which are Pfizer-BioNTech, AstraZeneca and Sinovac. The decline of the respective vaccines is being illustrated in Figure 5 (a). From Figure 5 (b), we observed that Sinovac shows the highest number of infected cases achieved compared to the other vaccines. About 176% increase in the number of highest infected cases calculated from the Pfizer-BioNTech while AstraZeneca only shows about 28% increases. Furthermore, on the last day of the simulations (300th day) in Figure 5 (c), Sinovac has increased up to 133% in the number of cumulative deaths while AstraZeneca has slightly increased to 8% from the number of cumulative deaths of Pfizer.
In conclusion, a COVID booster shot might be needed for Sinovac recipients to provide an extra layer of protection due to the faster waning period. These results also have helped to shape the current booster recommendations in Malaysia

CONCLUSION

From all the data obtained, we can conclude that getting vaccinated against COVID-19 can continue to reduce the risk of infections and deaths. Despite the vaccine effectiveness decreasing over time, the problem is still solvable by registering COVID booster shots for fast-waning vaccines. Besides the vaccination, there are other ways that can help to reduce the spread of the disease which is by implementing various countermeasures such as social distancing, wearing a mask in public and quarantining. On the other hand, vaccination also has been proven as one of the primary methods to achieve herd immunity in such a short period of time compare to natural immunity. Therefore, herd immunity can be achieved if at least 80% to 90% of the total population are vaccinated and only the risk of virus mutation or the emergence of variants that partly evade vaccine is negligible.

[1]. M. (n.d.). GitHub - MoH-Malaysia/covid19-public: Official data on the COVID-19 epidemic in Malaysia. Powered by CPRC, CPRC Hospital System, MKAK, and MySejahtera. GitHub. Retrieved September 20, 2021, from https://github.com/MoH-Malaysia/covid19-public#readme
[2] Fosu, G. O., Akweittey, E., & Adu-Sackey, A. (2020). Next-generation matrices and basic reproductive numbers for all phases of the Coronavirus disease. Open Journal of Mathematical Sciences, 4(1), 261–272. https://doi.org/10.30538/oms2020.0117
[3] C. (2021, November 25). After Three Months, Sinovac Vaccine Only 28% Effective Against ICU Admission. CodeBlue. Retrieved November 26, 2021, from https://codeblue.galencentre.org/2021/11/24/after-three-months-sinovac-vaccine-only-28-effective-against-icu-admission/
[4] Iacobucci, G. (2021, August 25). Covid-19: Protection from two doses of vaccine wanes within six months, data suggest. The BMJ. Retrieved November 11, 2021, from https://www.bmj.com/content/374/bmj.n2113
[5] Hualei Xin, Yu Li, Peng Wu, Zhili Li, Eric H Y Lau, Ying Qin, Liping Wang, Benjamin J Cowling, Tim K Tsang, Zhongjie Li, Estimating the Latent Period of Coronavirus Disease 2019 (COVID-19), Clinical Infectious Diseases, 2021;, ciab746, https://doi.org/10.1093/cid/ciab746
[6] Gallagher, B. J. (2020, May 1). Coronavirus: How long does it take to recover? BBC News. Retrieved October 10, 2021, from https://www.bbc.com/news/health-52301633
[7] Leoni, M. L. G., Lombardelli, L., Colombi, D., Bignami, E. G., Pergolotti, B., Repetti, F., Villani, M., Bellini, V., Rossi, T., Halasz, G., Caprioli, S., Micheli, F., & Nolli, M. (2021). Prediction of 28-day mortality in critically ill patients with COVID-19: Development and internal validation of a clinical prediction model. PLOS ONE, 16(7), e0254550. https://doi.org/10.1371/journal.pone.0254550

SIN3015 Mathematical Science Project

On Quadratic residues and a Conjecture of Sárközy

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Semester I, Academic Session 2021/2022

UNIVERSITI MALAYA

1. Introduction and objective

- A typical problem in **Additive Combinatorics**: Given an additive assumption about a set, we study the structure of the set.
- Sárközy's conjecture is one such problem.
- We shall **survey the progress made towards Sárközy's conjecture**.

2. Formulation of the problem

Recall the definition of a sumset.

- Let $\mathbb{F}_n$ be the finite field of n elements. Let A, B be subsets of an additive abelian group $G \subseteq \mathbb{F}_n$, then the **sumset** of A and B is defined as

$$A + B = \{a + b \mid a \in A, b \in B\}.$$

In 2012, Sárközy introduced the following generalisation.

- If $A_1, A_2, \ldots, A_k \subseteq \mathbb{F}_n$ and $|A_i| \geq 2$ for all $i = 1, 2, \ldots, k$, such that

$$A_1 + A_2 + \cdots + A_k = B \subseteq \mathbb{F}_n,$$

then $A_1 + A_2 + \cdots + A_k$ is said to be a (nontrivial) k-**decomposition** of B.

3. Definitions

- Let p be an odd prime. If the quadratic congruence $x^2 \equiv a \pmod{p}$ has a solution, then the integer a is a **quadratic residue** mod p. Otherwise, a is a **quadratic nonresidue** mod p.
- The **Legendre symbol**, $\left(\frac{a}{p}\right)$ is 1 if a is a quadratic residue mod p, -1 if a is a quadratic nonresidue mod p, and 0 if a is divisible by p.
- Let G be a finite abelian group. A **character** f of G is a group homomorphism from G to the multiplicative group of the complex field $\mathbb{C}$.
- Let G be the group of reduced residue classes mod n. The **Dirichlet character** mod n is an arithmetic function $\chi = \chi_f$ for each character f of G such that

$$\chi(a) = \begin{cases} f(\hat{a}), & \text{if } \gcd(a, n) = 1; \\ 0, & \text{if } \gcd(a, n) \neq 1. \end{cases}$$

Here, $\hat{a}$ denotes the residue class of a modulo n.
- **The Legendre symbol is a special case of the Dirichlet characters.**

4. Sárközy's conjecture

Let p be a large enough odd prime and let $\mathbb{F}_p$ be the finite field of p elements. Let $Q_p \subseteq \mathbb{F}_p$ denote the set of quadratic residues modulo p. Then there are no sets $A, B \subseteq \mathbb{F}_p$ with $|A|, |B| \geq 2$ such that

$$A + B = Q_p.$$

That is, the set Q_p **has no nontrivial** 2-**decomposition** for large enough prime p.

5. Main strategy

- Assume that a 2-decomposition of Q_p exists, find bounds for the cardinalities of the sumset.
- The crux of most proofs rest on bounds of character sums.
- A common and crucial tool used is the **Weil bound**.
- Other tools to consider are algebraic results since Q_p is a multiplicative subgroup of $\mathbb{F}_p$, as well as results for complex functions as group characters are complex-valued functions.

6. Lemma (Weil bound)

Let χ be a multiplicative character of order $d > 1$ of $\mathbb{F}_p^*$, the multiplicative group of the finite field of $\mathbb{F}_p$. Assume that the polynomial $g(x) \in \mathbb{F}_p[x]$ has S distinct zeros in the algebraic closure of $\mathbb{F}_p$ and $g(x)$ is not a constant multiple of the d-th power of a polynomial over $\mathbb{F}_p$. Then

$$\left| \sum_{x \in \mathbb{F}_p} \chi(g(x)) \right| \leq (S - 1)\sqrt{p}.$$

7. The case for A=B has been solved (Shkredov, 2014)

Using the Weil bound, the Cauchy-Schwarz inequality, and other results for complex functions, Shkredov obtained the following: Let p be an odd prime. Let $A \subseteq \mathbb{F}_p$ and Q_p be the set of quadratic residues mod p. If $A + A = Q_p$, then $p = 3$ and $A = \{2\}$.

8. Latest progress towards the conjecture (Chen & Yan, 2021)

Using the Weil bound and the Cauchy-Davenport Theorem, Chen & Yan showed the following: Let p be an odd prime. If the set of quadratic residues modulo p has a nontrivial 2-decomposition $A + B = Q_p$ with $|A|, |B| \geq 2$, then

$$\frac{7 - \sqrt{17}}{16}\sqrt{p} + 1 \leq |A|, |B| \leq \frac{7 + \sqrt{17}}{4}\sqrt{p} - 6.63.$$

9. Related problems: k-decompositions of Q_p

- The case for $k = 3$ was first solved by Sárközy for large enough prime p.
- Chen & Yan improved Sárközy's result by removing this requirement:
- For any odd prime p, the set Q_p has no nontrivial 3-decomposition

$$A + B + C = Q_p$$

with $|A|, |B|, |C| \geq 2$.

10. Related problems: 2-decompositions of the set of primitive roots mod p

- In 2013, Dartyge & Sárközy conjectured that **the set of primitive roots mod** p **has no nontrivial** 2-**decomposition** for large enough prime p.
- Although Dartyge & Sárközy (2013) and Shparlinski (2013) obtained parital results for this conjecture that are similar to that of the 2-decomposition of Q_p, it is still unsolved.

Main references

[1] Y.G. Chen and X.H. Yan, *A conjecture of Sárközy on quadratic residues*, J. Number Theory **229** (2021), 100–124.

[2] A. Sárközy, *On additive decompositions of the set of quadratic residues modulo p*, Acta Arith. **155** (2012), 41–51.

[3] I.D. Shkredov, *Sumsets in quadratic residues*, Acta Arith. **164** (2014), 221–243.

[4] I.E. Shparlinski, *Additive decompositions of subgroups of finite fields*, SIAM J. Discrete Math. **27** (2013), 1870–1879.

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UNIVERSITY OF MALAYA

Institute of Mathematical Sciences

PERFORMANCE ANALYSIS OF GEOMETRIC MEAN METHOD IN DETERMINING THE PEAK JUNCTION TEMPERATURE OF SEMICONDUCTOR DEVICE

"In electronics, semiconductor devices are extensively used and are stressed for reliability and performance. "

Introduction

- High performance electronics generate excessive junction temperatures, which compromise the performance of device
- To prevent device failure, high reliability users of microelectronic devices have been derating junction temperature
- Therefore, detailed calculations and simulations must be carried out to optimized the reliability

Objectives

1. To apply the Crank-Nicolson (CN) scheme to discretize a 1D heat conduction equation into a linear system of equation
2. To validate the performance of Geometric Mean (GM) method in solving the generated linear system from 1D heat conduction equation
3. To compare the efficiency of the proposed CN-GM method with CN-GS

1-D Heat Conduction Equation

Given $$K\frac{\partial^2 T}{\partial x^2} = pc\frac{\partial T}{\partial t} \quad ,0 < x < L,\ t > 0$$

The given initial condition: $T(x,0) = 294$ **Kelvin**

Boundary conditions: $$SK\frac{\partial T}{\partial x}\bigg|_{x=0} = -P_{in} \quad , \quad T(L,t) = T_{in} \quad , \quad t > 0$$

$K = 1.54$
$pc = 1.63$
$T_{in} = 300.15$ Kelvin
$P_{in} = 200$W

where,

T = Absolute temperature
K = Thermal conductivity
S = Surface area
L = Thickness of vertical device
p = Mass density of silicon
c = Specific heat of silicon
P_in = Input power
T_in = Input temperature

(Ghaffar et. al., 2008)

Discretization Scheme

For this part, CN scheme will be used to discretize the previous heat conduction problem. The derivative approximation are as shown below:

First order derivative: $$\frac{\partial T}{\partial t} = \frac{T_i^{j+1} - T_i^j}{\Delta t}$$

Second Order Derivative:

$$\frac{\partial^2 T}{\partial x} = \frac{1}{2}\left(\frac{T_{i-1}^j - 2T_i^j + T_{i+1}^j}{(\Delta x)^2} + \frac{T_{i-1}^{j+1} - 2T_i^{j+1} + T_{i+1}^{j+1}}{(\Delta x)^2}\right)$$

After substituting into the heat equation and boundary, a linear system will be obtained,

$$-\alpha T_{i-1}^{j+1} + (1+2\alpha)T_i^{j+1} - \alpha T_{i+1}^{j+1} = \alpha T_{i-1}^j + (1-2\alpha)T_i^j + \alpha T_{i+1}^j$$

where $\alpha = \frac{K(\Delta t)}{2pc(\Delta x)^2}$

The equation will then be expressed for each i = 1, 2, 3,..., n-1. Then a matrix representation of the linear system is obtained

where

$$A = \begin{bmatrix} 1+\alpha & -\alpha & 0 & \cdots & 0 \\ -a & 1+2\alpha & -\alpha & \ddots & \vdots \\ 0 & -\alpha & 1+2\alpha & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -\alpha \\ 0 & \cdots & 0 & -\alpha & 1+2\alpha \end{bmatrix}$$

$$T = \begin{bmatrix} T_1^{j+1} \\ T_2^{j+1} \\ \vdots \\ \vdots \\ T_{N-2}^{j+1} \\ T_{N-1}^{j+1} \end{bmatrix} \qquad B = \begin{bmatrix} \alpha T_0^j + (1-2\alpha)T_1^j + \alpha T_2^j + \alpha P_{in}\left(\frac{\Delta x}{SK}\right) \\ \alpha T_1^j + (1-2\alpha)T_2^j + \alpha T_3^j \\ \vdots \\ \vdots \\ \alpha T_{N-3}^j + (1-2\alpha)T_{N-2}^j + \alpha T_{N-1}^j \\ \alpha T_{N-2}^j + (1-2\alpha)T_{N-1}^j + \alpha T_N^j + \alpha T_{in} \end{bmatrix}$$

Iterative Method

In this section, Geometric Mean (GM) method will be used to solve the previous linear system

GM consists of solving two independent systems i.e.,T^1 and T^2

By splitting the matrix A,

A = D - L - U

- D is diagonal matrix
- -L is strictly lower triangular matrix
- -U is strictly upper triangular matrix

Form the general formulation,

$$\begin{cases} (D - \omega L)T^1 = [(1-\omega)D + \omega U]T^{(k)} + \omega B \\ (D - \omega U)T^2 = [(1-\omega)D + \omega L]T^{(k)} + \omega B \\ T^{(k+1)} = (T^1 \circ T^2)^{\frac{1}{2}} \end{cases}$$

The value of weighted parameter, ω is between $0 < \omega < 2$ and is chosen by trial-and-error process

(Muthuvalu & Sulaiman, 2008)

Results

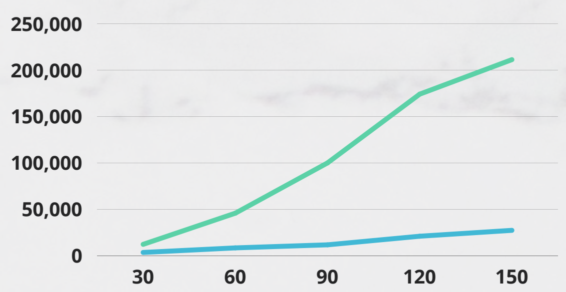

Figure 1 Comparison in terms of number of iterations

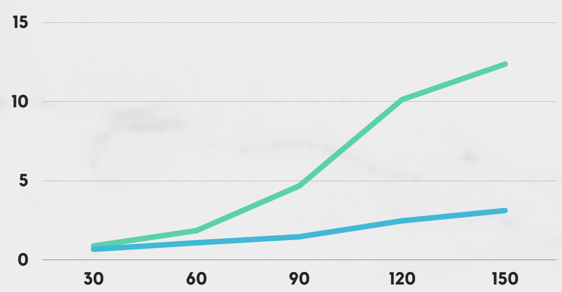

Figure 2 Comparison in terms of execution time (s)

Trendline for when t = 0.002, 0.006, and 0.010

For all cases of t=0.002, 0.006 and 0.010, with different mesh grid sizes

- The number of iterations of the GM method is reduced compared to GS method
- The execution time of the GM method is less compared to GS method
- The maximum temperature for both the methods is the same

Numerical Simulation

- Simulations are conducted for mesh grid sizes 30, 60, 90, 120, and 150
- Also, different elapsed time will be considered, t = 0.002, 0.006, and 0.010
- Gauss-Seidel (GS) is used as control method
- Convergence criteria, $\varepsilon = 10^{-10}$

Conclusion

1. Numerical approach based on CN scheme has been successfully implemented to the 1-D heat conduction equation
2. GM method greatly reduced the number of iterations and execution time when determining peak junction temperature
3. The proposed CN-GM method outperformed the CN-GS method in solving the linear system

References

- Abdul Ghaffar, Z. S., Alias, N., Sham Ismail, F., Mohamed Murid, A. H., & Hassan, H. (2008). Sequential algorithm of parabolic equation in solving thermal control process on printed circuit board. Malaysian Journal of Fundamental and Applied Sciences, 4(2). https://doi.org/10.11113/mjfas.v4n2.46
- Muthuvalu, M. S., & Sulaiman, J. (2008). Half-Sweep Geometric Mean Method for Solution of Linear Fredholm Equations. MATEMATIKA: Malaysian Journal of Industrial and Applied Mathematics, 24, 75–84. https://doi.org/10.11113/matematika.v24.n.224

A field is one of algebraic structure which consists of a set of elements where the operations such as addition, subtraction, multiplication, and division satisfy certain properties. The best example is the real numbers, along with the fields of rational numbers and complex numbers. These are all infinite fields as each contains an infinite number of distinct elements. Certain finite sets also satisfy the field properties when assigned appropriate operations. These finite fields and its properties are the focus of our discussion here.

Finite Field & Its Properties

Abstract Algebra

Group

We say G be a group of nonempty set with a binary operation $\bullet$ that assigns each ordered pair (a, b) of elements of G an element in G, which we denote as ab, satisfying properties:

1. *Closure*: If $a, b \in G$, then $a \bullet b$ also in G.
2. *Associativity*: For all $a, b, c \in G$, $a \bullet (b \bullet c) = (a \bullet b) \bullet c$.
3. *Identity*: There exist an element $e \in G$ (identity element) such that $ae = a = ea$ for all $a \in G$.
4. *Inverses*: If $a \in G$, there is an inverse element $a^{-1} \in G$, such that $a \bullet a^{-1} = 1 = a^{-1} \bullet a$.

A group G is abelian if:

5. *Commutativity*: For all $a, b \in G$, $a \bullet b = b \bullet a$.

Ring

Definition 1.1.1. We say R be a ring of nonempty set with a binary o[illegible] .. axioms:

For all $a, b, c \in R$

1. *Closure for addiction*: If $a \in R$, then $a + b \in R$.
2. *Associative addition*: $a + (b + c) = (a + b) + c$.
3. *Commutative addition*: $a + b = b + a$.
4. *Additive Identity or Zero Element*: There is an element 0_R in R such that $a + 0_R = a = 0_R + a$ for every $a \in R$.
5. For each $a \in R$, there exists an additive inverse, $-a$, such that $a + (-a) = 0$.
6. *Closure for multiplication*: If $a \in R$ and $b \in R$, then $ab \in R$.
7. *Associativemultiplication*: $a(bc) = (ab)c$.
8. *Distributive laws*: $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$.

R is commutative ring satisfying the axiom below:

9. *Commutative multiplication*: $ab = ba$ for all $a, b \in R$.

R is a ring with identity that contains the element 1_R satisfying the axiom below:

10. *Multiplicative identity*: $a1_R = a = 1_R a$ for all $a \in R$.

Field

A field is a set F on which two binary operations, called addition and multiplication, are defined and which contains two distinguished elements 0 and e with 0 =/= e. The field, F is an abelian group with respect to addition having 0 as the identity element, and the elements of F that are not 0 form an abelian group with respect to multiplication having e as the identity element. The two operations of addition and multiplication are linked by the distributive law a(b + c) = ab + ac. The second distributive law (b + c)a = ba + ca follows automatically from the commutativity of multiplication. The element 0 is called the zero element and e is called the identity element, usually be denoted by 1

Finite Field & Its Properties

Every finite field has characteristics p for some prime p.

A finite field K has order p^n, where p is the characteristic of K and $n = [K : \mathbb{Z}_p]$.

Let K be a finite field. Then the order of the finite field K is p^m for some prime p and natural number $m > 0$.

Let K be a finite field. Then its multiplicative subgroup K^* is cyclic.

Let K be a field of order p^m. Then, each $a \in K$ is a zero of the polynomial $x^{p^m} - a$. In particular, if $a \neq 0$, then a is a zero of $x^{p^m - 1} - 1$.

Let K_1, K_2 be finite fields with $|K_1| = |K_2|$. Then $K_1 \cong K_2$

Polynomial Ring

A polynomial ring is a commutative ring formed from the set of polynomials in one or more indeterminates or variables with coefficients in another ring, or often a field.

Polynomial Ring Over Field

Division algorithm for F[x]

If $f(x) \neq 0, g(x) \neq 0$ and $f(x), g(x) \in R[x]$, then there exist unique polynomials $q(x), r(x) \in R[x]$ such that $f(x) = q(x)g(x) + r(x)$, where $r(x) = 0$, or deg $r(x) <$ deg $g(x)$. (The polynomials $q(x)$ and $r(x)$ are called respectively the quotient and remainder.)

Remainder Theorem

If $f \in R[X]$ and $a \in R$, then for some unique polynomial $q(X)$ in $R[X]$ we have

$$f(X) = q(X)(X - a) + f(a)$$

Hence $f(a) = 0$ if and only if $X - a$ divides $f(X)$.

Factor Theorem

Let F be a field, $a \in F$, and $f(x) \in F[x]$. Then a is a zero of $f(x)$ if and only if $x - a$ is a factor of $f(x)$.

Congruence & congruence classes

The concept of "congruence" may be thought of as a generalization of the equality relation. Two integers a and b are equal if their difference is 0 or, equivalently, if their difference is a multiple of 0. If n is a positive integer, we say that two integers are congruent modulo n if their difference is a multiple of n. To say that a - b = nk for some integer k means that n divides a - b. So we have this formal definition:

Congruence in F[X]

The concept of congruence of integers depends only on some basic facts about divisibility in Z. If F is a field, then the polynomial ring F[x] has essentially the same divisibility properties as does Z. So it is not surprising that the concept of congruence in Z and its basic properties can be carried over to F[x]

Extension Field

Field extensions are fundamental in algebraic number theory and in the study of polynomial roots through Galois theory and are widely used in algebraic geometry.

Let E and F be fields. We say field E is an extension field of field F if $F \subseteq E$ and the operation of F are those of E restricted to F. We write E is an extension field of F as $F \leq E$ or E/F.

Finite Extension

Let the field E be an extension field of the field F, E/F. If the dimension of E as a vector space over F is finite, then E is said to be a finite extension of F.

The dimension of E as a vector space over F is called the degree of E over F and we write it as $[E : F]$.

Algebraic Extension

An extension field E of a field F is said to be an algebraic extension of F if every element of E is algebraic over F.

The Investigation On The Relationship Between Solar Activity and Large Earthquake Worldwide

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Introduction

- There are quite a lot of earthquakes have been recorded worldwide until now.
- Some of the largest earthquakes recorded were in Chile (1960) and in Alaska (1964) which both were extremely powerful and very destructive.

Objectives

1. To understand the natural phenomenon of solar activity and earthquake.
2. To investigate a complex relationship between solar and global seismicity.
3. To validate the theory that there exists a strongly significant link between solar activity and large earthquake worldwide.

Problem Statements

- Some authors believed the earthquake occurrences is connected to the geomagnetic storms (Nur Hidayah Ismail et al., 2021).
- Rudolf Wolf, an astronomer, proposed that sunspots may impact earthquake occurrence.
- Since then, many scientists behind this idea believed that sunspots are responsible for the increases in activity on Earth.
- Some authors claimed that they were unable to determine whether there exists an interaction between solar-terrestrial parameters and earthquake occurrences (Love & Thomas, 2013).

Scope of Study

1. Earthquake worldwide
2. 1st of August 1996 – 31st of August 2008

Data

Earthquake worldwide

- ISC-GEM catalogue
- 5.6 is the minimum magnitude
- six magnitude threshold: M ≥ 5.6, 6.0, 6.5, 7.0, 7.5, 8.0

- Represent as earthquake data

Proton measurements

- SOHO satellite
- Proton variables: density ρ, velocity v, flux ρv and dynamic pressure $\rho v^2/2$

- Represent as solar activity

Methodology

- We divided it into 5 conditions, given a threshold V_T:

Conditions	Description
1. aT	Days where V above the V_T threshold
2. 2lstDy aT	Second to last day where V above the V_T threshold
3. lstDy aT	Last day where V above the V_T threshold
4. 1Dy bT	First day where V below the V_T threshold
5. 2Dy bT	Second day where V below the V_T threshold

- Firstly, we interpret the non-dimensional average of V, V_{av_ad} as

$$V_{av\_ad} = \frac{V_{av} - V_{min}}{V_{max} - V_{min}}$$

- Then, we verify a varying threshold, V_T as

$$V_T = V_{min} + V_{step}(V_{max} - V_{min})$$

- We assign V_{step} ranges from V_{av_ad} to 1, with steps of 0.01.
- Event relative rate R for each V_T, with a given condition C is

$$R = \frac{\frac{E_C}{D_C}}{\frac{E - E_C}{D - D_C}}$$

where E_C is the number of events occurred that satisfies the condition C and D_C is the number of days that satisfies the condition C. Whereas E and D is the total number of events that occurred in those days and the number of SOHO available days.

- Hence, by these three equations, we can construct plots of R versus V_{step} for each variable V.

Results

- When R *oscillates* around 1, we can infer that there is an earthquake occurred with respect to the proton variables, V.
- Clearly, we can see there is a condition (1st day below threshold) where the R *values* oscillate around 1 as illustrated in the plot for proton density and proton flux.
- Therefore, we can conclude that there is a correlation between proton variables and global seismicity where earthquake occurs during the first day after the variable decreases below the threshold value (1Dy bT).

Conclusion

In conclusion, we have proven the existence of a strongly significant relation between solar activity and large earthquake worldwide. Moreover, we can conclude that the occurrence of large earthquake worldwide happened during the first day below a certain proton density threshold (1Dy bT).

Reference

1. Nur Hidayah, I., Nazhatulshima, A., Nur Anisah, M., & Mohammad Redzuan, T. (2021). Analysis of Geomagnetic A_p Index on Worldwide Earthquake Occurrence using the Principal Component Analysis and Hierarchical Cluster Analysis. *Sains Malaysiana*, 50(4), 1157-1164. Retrieved from http://doi.org/10.17576/jsm-2021-5004-25
2. Love, J. J., & Thomas, J. N. (2013). Insignificant solar-terrestrial triggering of earthquakes. *Geophysical Research Letters*, 40(6), 1165-1170. doi:10.1002/grl.50211

SIN3015 MATHEMATICAL SCIENCE PROJECT

ROOK POLYNOMIALS IN THREE DIMENSION

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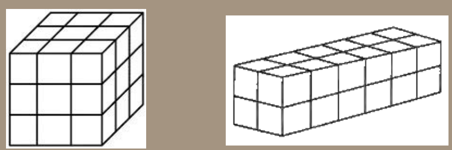
Supervised by : Dr Wong Kok Bin

INTRODUCTION

Let C be a board of size m. For any $k \leq m$, let $r_k(C)$ denote the number of ways of placing k non-attacking rooks on C. The generating function for $r_k(C)$ is

$$R(x, C) = r_0(C)x^0 + r_1(C)x^1 + r_2(C)x^2 + \ldots + r_m(C)x^m$$



Figures 1: Full boards.

- A $m_1 \times m_2 \times m_3$ full board has subsets of $\{1, 2, \ldots, m_1\} \times \{1, 2, \ldots, m_2\} \times \{1, 2, \ldots, m_3\}$.
- A non-full board is a board with any missing/restricted cell.
- Cell (i, j, k) is refer to positions where rooks can be placed.
- The i, j and k correspond to slabs, walls and layers respectively with $1 \leq i \leq m_1$, $1 \leq j \leq m_2$, $1 \leq k \leq m_3$.
- When a rook is placed in a cell, no longer another rook can be placed in the same wall, slab, or layer.

FAMILY OF THREE DIMENSIONAL BOARDS

1. Triangle Board

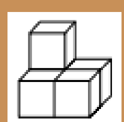


Figure 2: Size 2 of Triangle board.

- In general, there is a $(m+1) \times (m+1)$ layer at the bottom of a size m triangle board.
- Property: There is only one way to place m rooks on a size m triangle board.

Theorem. (Triangle Board). The number of ways to place k non-attacking rooks on a size m triangle board in three dimension is equal to

$$T(m+1, m+1-k), \text{ where } 0 \leq k \leq m.$$

- The numbers turn out to be the central factorial numbers defined recursively by

$$T(n, k) = T(n-1, k-1) + k^2 \cdot T(n-1, k)$$
$$\text{with } T(n, 1) = 1 \text{ and } T(n, n) = 1$$

2. Genocchi Board

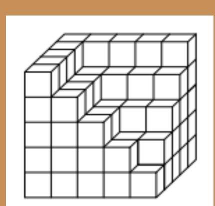


Figure 3: Size 5 of Genocchi board.

- The complement of size m Genocchi board is just size $m-1$ triangle board.

Theorem. (Genocchi Boards). The number of ways to place m non-attacking rooks on a size m Genocchi board is the unsigned $(m+1)^{th}$ Genocchi number.

Table 4: Rook polynomials for various sizes of this board.

m	0	1	2	3	4	5	6	7
G_{m+1}	-1	1	-3	17	-155	2073	-38227	929569
r_m(unsigned G_{m+1})	1	1	3	17	155	2073	38227	929569

G_{m+1} is from sequence A001469 and r_m is from sequence A110501 in [Sloane 2009].

PROPERTIES OF ROOK POLYNOMIAL IN THREE DIMENSION

Theorem 1. The number of ways of placing k non-attacking rooks on the full $m_1 \times m_2 \times m_3$ board is equal to

$$\binom{m_1}{k}\binom{m_2}{k}\binom{m_3}{k}(k!)^{3-1}$$

Theorem 2. (Disjoint Board Decomposition). If A and B be boards with no interfering wall, slab or layer , then the rook polynomial for $A \cup B$ is the product of the rook polynomials for the two parts which is

$$R_{A \cup B}(x) = R_A(x)R_B(x)$$

Theorem 3. (Cell Theorem). Let B be a board and let s be a cell in B. Suppose B' is the board obtained by deleting s and every cell in the same wall, slab and layer corresponding to s from B, while B'' is the board obtained from B by deleting only s. Then

$$R_B(x) = xR_{B'}(x) + R_{B''}(x).$$

Theorem 4. (Complementary Board Theorem). Let $\bar{A}$ be the complement of A which $\bar{A}$ is a restricted board with size n cube. Then the number of ways that we can place k non-attacking rooks on A with respect to the restrictions is equal to

$$\sum_{k=0}^{n}(-1)^k (n-k)!^{3-1} r_k(\bar{A})$$

where $r_k(\bar{A})$ is the number of ways of placing k non-attacking rooks on the board $\bar{A}$ of the forbidden positions.

GENERAL PROBLEM

- There is a situation where five people entering a restaurant with their own unique hat and coat.
- We are interested in the number of ways that the five people can leave the restaurant without both of their original items.
- Relate the situation with $5 \times 5 \times 5$ cube and let each slab, wall and layer represent the coat, hat and person respectively.
- Place the restrictions along the main diagonal.

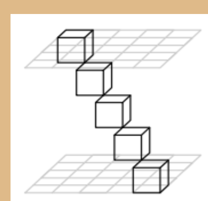


Figure 5: The 5 cubes are cells (i, i, i) representing the restrictions.

The rook polynomial for each restriction is $1 + x$, since there are 5 restrictions and all are disjoint, by applying Disjoint Board Decomposition Theorem we get $(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$.

Using Complementary Board Theorem the number of ways for the five people leave restaurant without both of their original items is $(5!)^2(1) - (4!)^2(5) + (3!)^2(10) - (2!)^2(10) + (1!)^2(5) - (0!)^2(1) = 11844$.

MAIN REFERENCES

Alayont, F., & Krzywonos, N. (2013a). Rook polynomials in three and higher dimensions. *Involve, a Journal of Mathematics*, *6*(1), 35–52. https://doi.org/10.2140/involve.2013.6.35

Krzywonos, N. (2009). *Rook Polynomials In Higher Dimensions*. ScholarWorks@GVSU. https://scholarworks.gvsu.edu/sss/29/

A Study of Jordan Canonical Forms of Linear Operators

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ABSTRACT

In this study we will study Jordan canonical forms of linear operators on finite dimensional vector spaces over fields. A review of some preliminary tools and important results that are related to the project will be conducted. By using these results, we will study the existence of the Jordan canonical forms of linear operators on finite dimentional vector spaces over fields. We then proceed to study the uniqueness of Jordan canonical form.

EXISTENCE OF JORDAN CANONICAL FORM

Let $\psi : V \to V$ be a linear operator. Suppose that $\mathcal{B}$ is an ordered basis of V such that

$$[\psi]_{\mathcal{B}} = \begin{pmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_k \end{pmatrix} = J_1 \oplus J_2 \oplus \cdots \oplus J_k,$$

where each J_i is a square matrix of the form

$$J_i = \begin{pmatrix} \lambda & 1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda & 1 \\ 0 & 0 & 0 & \cdots & 0 & \lambda \end{pmatrix} \quad \text{for some eigenvalue } \lambda \in \mathbb{F} \text{ of } \psi.$$

The matrix J_i is called a **Jordan block** corresponding to λ and the matrix $[\psi]_{\mathcal{B}}$ is called a **Jordan canonical form** of ψ.

We also say that the ordered basis $\mathcal{B}$ is a **Jordan basis** of ψ.

DEFINITION

Let $\psi : V \to V$ be a linear operator and let $\lambda \in \mathbb{F}$ be an eigenvalue of ψ. The **generalized eigenspace** of ψ corresponding to λ, denoted by $K_{\psi,\lambda}$, is the subset of V defined by

$$K_{\psi,\lambda} = \{v \in V : (\psi - \lambda I)^p(v) = 0 \text{ for some positive integer } p\}.$$

LEMMA 1

Let $\psi : V \to V$ be a linear operator such that the characteristic polynomial of ψ splits. If $\lambda \in \mathbb{F}$ be an eigenvalue of ψ with algebraic multiplicity m, then the following holds.

(i) $K_{\psi,\lambda}$ is a ψ-invariant subspace of V.

(ii) $K_{\psi,\lambda} = Ker((\psi - \lambda I)^m)$.

LEMMA 2

Let $\psi : V \to V$ be a linear operator such that the characteristics polynomial of ψ splits over $\mathbb{F}$. Let $\lambda_1, \dots, \lambda_k \in \mathbb{F}$ be all the distinct eigenvalues of ψ. Then

$$V = K_{\psi,\lambda_1} + \cdots + K_{\psi,\lambda_k}$$

LEMMA 3

Let $\psi : V \to V$ be a linear operator and let $\lambda \in \mathbb{F}$ be an eigenvalue of ψ. An ordered set $\{x_1, \dots, x_k\}$ of vector in V is said to be a **Jordan chain** of ψ corresponding to λ provided that

$$\psi(x_1) = \lambda x_1,$$
$$\psi(x_i) = \lambda x_i + x_{i-1} \qquad \text{for } i = 2, \dots, k.$$

LEMMA 4

Let $\psi : V \to V$ be a linear operator such that the characteristic polynomial of ψ splits over $\mathbb{F}$. Let $\lambda_1, \dots, \lambda_k \in \mathbb{F}$ be all the distinct eigenvalues of ψ with corresponding algebraic multiplicity of $m_1, \dots, m_k$, respectively. If $\mathcal{B}_i$ is an ordered basis for $K_{\psi,\lambda_i}, i = 1, \dots, k$, then the following statement hold.

(i) $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset$ for all distinct pair of integers $1 \le i, j \le k$.

(ii) $\mathcal{B}_1 \cup \cdots \cup \mathcal{B}_k$ is an ordered basis for V.

(iii) $\dim K_{\psi,\lambda_i} = m_i$ for $i = 1, \dots, k$.

LEMMA 5

Let $\psi : V \to V$ be a linear operator and let $\lambda \in \mathbb{F}$ be an eigenvalue of ψ. Then the generalized eigenspace $K_{\psi,\lambda}$ of ψ corresponding to λ has an ordered basis $\mathcal{B} = \mathcal{B}_1 \cup \cdots \cup \mathcal{B}_m$ consisting a union of disjoint **Jordan chains** $\mathcal{B}_1, \dots, \mathcal{B}_m$ of ψ corresponding to λ corresponding to λ and

$$[\psi|_{K_{\psi,\lambda}}]_{\mathcal{B}} = J_1 \oplus \cdots \oplus J_m,$$

where each J_i is a Jordan block corresponding to λ.

THEOREM 1

Let V be a finite-dimensional vector space over a field $\mathbb{F}$ and let $\psi : V \to V$ be a linear operator whose characteristic polynomial splits over $\mathbb{F}$. Then V has a basis which is a disjoint union of Jordan chains of ψ, i.e., V has a Jordan basis for ψ.

Uniqueness of Jordan Canonical Form

THEOREM 2

Let V be a finite-dimensional vector space over a field $\mathbb{F}$. Let $\psi : V \to V$ be a linear operator and let $\lambda \in \mathbb{F}$ be an eigenvalue of ψ. If V has a Jordan basis which is a disjoint union of Jordan chains of ψ, then the number of Jordan chains of length m of ψ corresponding to λ is

$$(\dim Ker(\psi - \lambda I)^m - \dim Ker(\psi - \lambda I)^{m+1}$$
$$+ (\dim Ker(\psi - \lambda I)^m - \dim Ker(\psi - \lambda I)^{m-1}).$$

References

[1] W.L. Chooi, Linear Algebra: Lecture Note, Institute of Mathematical Sciences, Universiti Malaya, Kuala Lumpur, 2020.

[2] S.H. Friedberg, A.J. Insel, L.E. Spence, Linear Algebra, Fourth edition, Prentice-Hall, Inc., New Jersey, 2003.

[3] , R. Larson, Elementary Linear Algebra, seventh edition, Brooks/Code Cengage Learning, 2013.

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MATHEMATICAL SCIENCE PROJECT

STIRLING NUMBER OF THE SECOND KIND

Definition

Stirling number of the second kind is the number of ways to partition n labelled elements into k unlabelled nonempty sets.
It is denoted by $S(n,k)$.

Explicit formula

For $n \geq 0$ *and* $0 \leq k \leq n$

$S(n,k)$ can be expressed as

$$\frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$$

Bounds

For $1 \leq k \leq n-1$

Lower bound of $S(n,k)$

$$L(n,k)=\frac{1}{2}(k^2+k+2)k^{n-k-1}-1$$

Upper bound of $S(n,k)$

$$U(n,k)=\frac{1}{2}\binom{n}{k}k^{n-k}$$

Recurrence Relation

For all $0<k<n$

$$S(n,k)=S(n-1,k-1)+kS(n-1,k)$$

Inequalities

If $k>2$ *and*

$$(k-2)S(2k-3,k-1)>kS(2k-3,k)$$

then,

$$S(2k-2,k-1)>S(2k-2,k)$$

If $n \geq 4$ *and* $\frac{1}{2}n+1 \leq k \leq n-1$

then,

$$(n-k)S(n,k)>(k+1)S(n,k+1)$$

If $n \geq 4$ *and* $2 \leq k \leq \frac{1}{2}n,$

then,

$$S(n,k)>S(n,n-k+1)$$

Asymptotic Behaviour

When n *is large,*

$$k_n=\frac{n}{\log\ n}+O(n(\log\ \log\ n)^{\frac{1}{2}}(\log\ n)^{\frac{-3}{2}})$$

Table (Example)

n\k	1	2	3	4	5	6
1	1					
2	1	1				
3	1	3	1			
4	1	7	6	1		
5	1	15	25	10	1	
6	1	31	90	65	15	1

Relation

Bell number

$$B_n=\sum_{k=0}^{n}S(n,k)$$

Stirling number of the first kind

Reference

[1] Boyadzhiev, K.N.(2012). Close encounters with the Stirling numbers of the second kind. *Mathematics Magazine,85*(4),252-266. https://doi.org/10.4169.math.mag.85.4.252

[2] Brualdi, R.A.(2010). *Introductory Combinatorics Fifth Edition.* Pearson Prentice Hall

[3] *Counting and Stirling numbers.*(n.d.). Retrieved January 17,2022, from http://www.sfu.ca/ mdevos/notes/combstruct/stirling.pdf

[4] Dobson, A.J.(1968).A note on Stirling number of the second kind. *Journal of Combinatorial Theory,5*(2), 212-214. https://doi.org/10.1016/S0021-9800(68)80060-2

[5] Grassl, R., Levin, O.(n.d.). *More Discrete Mathematics via graph theory.* Stirling number of the Second Kind. Retrieved January 17,2022, from http://discrete.openmathbooks.org/more/mdm/secadv-stirling.html

[6] Weisstein, E.W.(n.d). *Stirling number of the Second Kind. from Wolfram MathWorld.* Retrieved January 17,2022, from https://mathworld.wolfram.com/StirlingNumberoftheSecondKind.html